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# THE MATHEMATICS TEACHER

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# THE MATHEMATICS TEACHER

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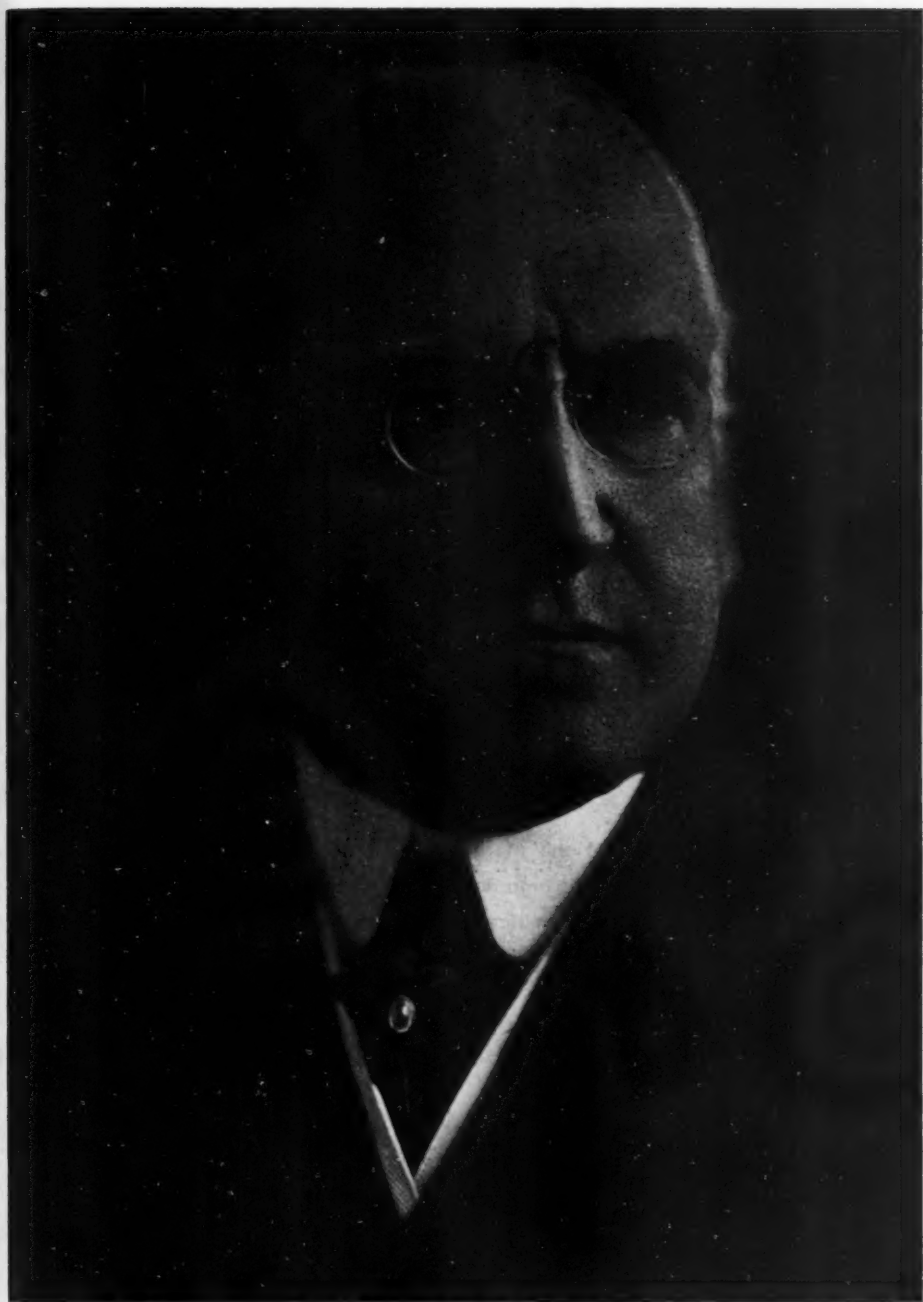
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# THE MATHEMATICS TEACHER

Volume XXXVII



Number 6

Edited by William David Reeve

## High School Mathematics After the War\*

By WALTER H. CARNAHAN

*Purdue University, Lafayette, Indiana*

WE ARE now engaged in a great global war which compels a searching evaluation of the educational principles that have been guiding us. If this evaluation were confined to the military results of the application of these principles, we could readily excuse any revealed shortcomings on the ground that we have been justifiably training for the pursuits of peace. But to the most casual observer, it is clear that the exposed defects are by no means wholly or even chiefly military. So far as the speaker is aware, not one of the many volatile critics of our educational products has complained because girls in overalls or boys in uniform have not been taught how to make bombs or to navigate bombers. The complaints are that they cannot understand simple directions of any kind written in simple English, that they do not understand simple arithmetic, that they cannot apply its principles to the solution of simple problems, that they cannot think clearly and independently. Before we enter into detailed discussion of these criticisms, let us hasten to credit these young people with unlisted scores of fine qualities and

real achievements which make us forever proud of them and of the educational system that has contributed to their development. But "pointing with pride" is not often a promising preliminary to productive planning, and consequently I shall invite your consideration of certain adjustments that improvement seems to demand.

May we, like Lowell's musing organist, begin "doubtfully and far away"? Twenty-five hundred years ago, mathematics was a subject for mature philosophers, Thales, Plato, Euclid, Archimedes, Pythagoras. When these men studied mathematics, it was not merely a case of philosophers considering mathematics, but philosophers investigating one of the chief branches of philosophy. For many hundreds of years, the subject remained on precisely this basis. Mathematics students were philosophers, accomplished scholars, often gray beards, creative research investigators.

During the middle ages in the church schools and universities, the situation changed but little. Creative research and scholastic achievement were doubtless at a lower level, but mathematics was still for mature men with philosophical minds. It was taught by scholars to those who would be scholars. As late as three hundred years ago, it could scarcely be regarded as a sub-

\* Read at a meeting of the Greater Cincinnati Mathematics Club and a sectional meeting of The National Council of Teachers of Mathematics on March 18, 1944.

ject of general interest. Consider that the classes of the great Isaac Newton sometimes had no more than three or four students.

Two hundred years ago in America, plane geometry was a university subject, that is to say, it was taught to gentlemen, scholars, doctors, lawyers, ministers, political leaders, and to almost no one else. Less than one hundred years ago, a prominent plane geometry text (Olney's) was designated as a university geometry.

Forty years ago, algebra and plane geometry having gradually come down to the secondary school level, were being taught to all high school pupils. But remember that even so late as that time, high schools were preparatory schools, existing to equip young men and women for entrance into college. And, generally speaking, this meant that algebra and geometry were still being taught to those who would be ladies and gentlemen, people of culture, ministers, teachers, lawyers, doctors. High school pupils were a very highly selected group, not necessarily on the basis of ability, but on the basis of purpose, ambition and plan. It is an old and familiar story, but let it be told once more as the speaker experienced it. In 1909, I finished the third high school year (all that was offered) in a village in southern Indiana. There was but one teacher for all subjects, and there were no electives. The whole high school and eighth grade, recitation room, study hall, library, and laboratory (and a big coal burning stove) were housed in one room the size of a large living room. This high school served a whole township. There were about twenty young ladies and gentlemen, all preparing for college and a profession.

Two years after finishing this high school, I returned as teacher. How things had changed! There were now two regular teachers and a part time music teacher. There were two rooms instead of one. The pupil enrollment had trebled. Not only the future professional men and women were there, but the blacksmiths, the farmers,

the storekeepers, the mechanics, the clerks, the miners, the housewives, and one boy whose expressed ambition was to be the man in the moon. How these people happened to be there is a long and significant story which you know well. Let me hasten to say that they have come to stay and that they are where they belong.

What now shall we say of the mathematics of philosophers, scholars, cultured gentlemen and ladies? While the mathematics of gray beards was in the process of being brought down to the level of the secondary schools and its fourteen year olds, we have been compelling the former highly selected fourteen year olds to move over and make room for Tom, Dick and Harry who used to quit school and go to work. Extended to other high school subjects, to science, to English, to Latin, this is the most far reaching educational change ever recorded in all the history of the world. We are just beginning to realize its complete significance.

After the first shock of the impact of Tom, Dick and Harry upon the high schools, we began to ask each other, "What can we do with them?" The war is revealing what we have been doing with them mathematically, and we are scarcely proud of it. It is the purpose of this paper to examine certain results of our work and try to point the way to improvement.

*We need a constantly emphasized program of skill maintenance.* The young men entering the armed services and the young women replacing them in shops and factories are deficient in the ability to use arithmetic readily and dependably. The trouble is not that they have not been taught and drilled; they have been instructed, but skills once acquired have been allowed to deteriorate. The teacher of algebra or general mathematics is conscious of this loss of skill in handling numbers, and often complains about it, but too frequently she does not take it as part of her responsibility to restore and maintain these skills. The teacher of geometry recognizes the additional loss of facility in algebra, but seldom takes the responsibility

of maintaining skills in both arithmetic and algebra. No skill is ever acquired as a permanent possession, whether it be in music or in mathematics. To be retained, it must be constantly practiced. Teachers of algebra and geometry should never discount this fact, and should willingly accept the responsibility for constant reteaching and drill on previously learned fundamentals. This relates to the first of the practical aims listed by the wise National Committee on Reorganization of Secondary mathematics in 1922. It was sound policy before 1922, and it will be sound policy after this war. It should be one of the *musts* of high school mathematics instruction. Textbook writers, school administrators, examining authorities, teacher training institutions, and teachers should concentrate on this as essential. Perhaps unified mathematics organization would have done much to insure this maintenance of skills, but unified mathematics seems to be making little headway, and we shall have to try other means for securing retention.

The need for maintenance of skills is not a war emergency problem. Employers have long complained of lost (or, as they supposed, unacquired skills). College instructors find the same defect. It is a permanent disease, but the cure does exist. It consists in making definite provision for constant restoration.

*We need constant emphasis on teaching for understanding.* We need a new comprehension of how understanding is attained. No doubt every high school teacher of mathematics believes that she emphasizes understanding, but one of the difficulties is that too often we believe that permanent understanding results from a single careful explanation, whereas for most learners this is not the case. For example, in teaching the addition of common fractions, the teacher will carefully and skilfully explain how to reduce the fractions to common denominators and give the reasons which justify the process. But too often this explanation is at once followed by the statement of a rule which is capable of being applied

without understanding. Pupils find the mechanical application of the rule quicker and less troublesome than the application of the fundamental principle, and too often they forget the principle and use the rule mechanically. "Divide the given denominator into the common denominator, multiply the given numerator by the result, and write the product over the common denominator." Asked if he understands this, the pupil will reply promptly that he does. As evidence of this, he can work a problem using the rule. In algebra he relearns the rule and goes on to make other mechanical applications of it. What is the result of this manipulation without understanding? Let me answer this question by giving an actual incident from the classroom of a technical university. Two sophomores in the school of metallurgical engineering were overheard in an argument concerning the addition of fractions. The problem was  $\frac{1}{2} + \frac{1}{3}$ . One said, "The result is  $\frac{2}{3}$ , because  $1+1=2$ , and  $2+3=5$ ." The other said, "It is  $\frac{5}{6}$ , because the way I remember it you have to multiply the numerators and the denominators." Neither of these young men understood the principle involved, and each was concerned only with trying to remember a rule. Would it not have been better if they had been taught to use the fundamental principle with understanding even though it is slower? It seems to the speaker that mathematics would come much nearer justifying its claimed value of teaching people to think if we should encourage application of fundamental principles rather than permit too early mechanical manipulation.

There can be no doubt that application of fundamental principles takes more time than application of short cut rules, but we need to keep in mind that mastery comes not from doing one hundred examples by rule so much as doing a fourth of that number with understanding. It seems to the speaker that in teaching high school mathematics after the war we should strive more and more for clear understanding. It is true that there does come a time when

the processes of mathematics should become automatic and be applied like the multiplication or addition combinations of arithmetic without rationalization, but this time almost certainly comes much later in learning experience than common teaching practice seems to indicate.

*The teacher of mathematics should be a teacher of reading.* Whenever two teachers of mathematics enter into a discussion of the deficiencies of pupils, the criticism is almost invariably heard that they do not know how to read. Very well, then, let us face it. Let us willingly become teachers of reading and learn methods for the teaching of reading in mathematics effectively. Of course, the regular teacher of reading might prepare pupils better than is now being done, but the fact is that reading is an ability that has to be relearned in connection with every subject in school, and the teacher of mathematics as well as the teacher of science or history must teach pupils how to read the subject. This is one of the steps in teaching for understanding.

*There should be a program of pre-graduation testing for retention of minimum essentials.* Very simply, this means that every senior should be tested before his last semester in school to find out whether he has retained the minimum essentials of arithmetic and algebra (and perhaps certain other subjects). If he has not retained these, he should be required to take and pass a course of remedial instruction in those subjects in which he is deficient. Education has little value when it becomes merely a matter of spending time and accumulating credits. The public has a right to expect that certain essential skills should be permanent.

A word of caution here: We should be clear about the meaning of *minimum essentials*. I am not recommending an achievement examination to determine how much the pupil knows, or whether he remembers all he has been taught, or what is his rank in class. I am merely recommending a brief test to see whether he is prepared to use as a tool certain funda-

mental skills that the educated citizen may reasonably be expected to be able to use.

*There should be a definite, continuous program of adjustment of mathematics instruction to the abilities of the learners.* Whether this adjustment is brought about by ability grouping, or by adjustment to individuals within the same class group, or by some other means, the interests of bright as well as of slow pupils demand this adjustment. In future we must find a way in our democratic system of education to develop to the full the abilities of the intellectually capable. They are still with us as they were when the high school existed for them alone, and one of our grave dangers is that they will be neglected, or worse, that they will be reduced by our system to mediocrity. Europeans educate to a high degree and exclusively these fortunate few. It would be in the nature of national suicide for us to fail to educate them effectively. This should be done, not to give them advantages over other Americans, but as a development of one of our national resources which can contribute to our strength to endure in the keenest international competition.

While we are educating these leaders, we must not neglect those others who are to be followers. It is our American strength to make full use of every human resource. A democracy has little other excuse for existence. The education of Tom, Dick and Harry is slow, expensive and often discouraging, but to neglect it would be disastrous to the ideals for whose realization our kind of government exists. Leaders and followers, we need them all trained as fully as their different capacities permit.

*The teacher of mathematics should be a salesman of the subject.* We teach a subject whose fascination and cultural and practical values are exceeded by no other subject. It would be a mistake to fail to reveal these qualities to pupils. Imagine a teacher of literature or history who failed to reveal the human interest and beauty of those subjects. The human interest and cultural value of mathematics are not often on the

surface, but they richly reward the effort required to reveal them. Every mathematics teacher should be thoroughly familiar with the materials to be found in such books as *Mathematics and The Imagination*, by Kasner and Newman, *Mathematics, Its Magic and Mastery*, by Bakst, *Men of Mathematics*, by Bell, or *Mathematical Recreations*, by Ball. As occasion permits, pass on the fascination of these materials to the pupils. Make use also of slides, motion pictures, wall charts, puzzles, and fallacies as a means of selling the subject. To some pupils, of course, the very evident power of the subject is sufficient to hold them, but this will not hold them all.

This discussion deals with only a few illustrative problems that we shall have to solve in post-war mathematics teaching. But these illustrations will have to suffice for the present. May I now direct a few suggestions to the teachers of the subject.

1. Let us keep alert to educational developments during these strenuous times by reading the published literature relating to our problems, by meeting together to exchange ideas, and by thinking care-

fully through the problems that confront us. Of course, we must collect scrap, sell stamps, knit, bake cookies, make posters, learn air raid precautions, support Red Cross, serve U.S.O., give blood, keep up on the news, and a dozen other obligations. But one peculiar obligation is ours, and that is to solve our country's educational problems, and to do this we need to meet and think together.

2. In your school, try to encourage a few capable boys and girls to become teachers. The teacher problem is soon going to be critical the nation over. Material rewards for teaching are little more attractive today than formerly, but the old appeal of service has not wholly lost its power. Try it.

3. When the war is over, let's go back to school. We are getting behind on ideas, instruction and inspiration. When victory comes, let's crowd the summer school classes in every university in the country. That should be our testimony of faith in the permanent value of education in a democracy.

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#### OCTOBER

"There comes a month in the weary year,  
A month of leisure and healthful rest;  
When the ripe leaves fall and the air is clear;  
October, the brown, the crisp, the blest."

October turned my maple leaves to gold;  
They most are gone now, here and there one lingers;  
Soon these will slip from out the twig's weak hold  
Like coin, between a dying miser's fingers.

—THOS. BAILEY ALDRICH

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#### Notice to Our Readers

DUE TO circumstances over which we have no control, the 18th yearbook is not yet ready for distribution, but it will be ready very soon. All orders will be filled as fast as possible after publication.

EDITOR.

# Basic Mathematics—A Key to Democracy

By RALPH MANSFIELD

Chicago Teachers College, Chicago, Ill.

MANY REASONS have been advanced for the need of a universal language. The latest development advanced to meet this need is Basic English—a language based on 850 words and with no less a proponent than the Prime Minister of England. Among the many reasons given for the need of a universal language, the principal one is that such a language offers a common medium for the exchange of thought the world over, and through such a medium it may be possible to introduce democracy to many beleaguered peoples.

As mathematicians, we are well aware of the universality of mathematical language, but somehow or other we have failed to appreciate the democratic aspects of mathematics. We might call this phase of mathematics "Basic Mathematics" and introduce it throughout our schools in order to clarify the meaning of democracy for our students. Before discussing Basic Mathematics, however, let us digress for a few minutes on present day trends in mathematics.

If we were to act as reviewers of high school and grade school mathematics texts we should see many fancy titles and pretty covers come to our desks. Each one would seem to carry some mute, inanimate plea to believe the words set forth in the preface. Only because of some individual skepticism would we set those pleas aside and delve into the textual material. It would make us very unhappy to realize, after perusal of the text, that in only a small number of cases was the skepticism unwarranted.

If a book were to come for review, entitled "Mathematics for Everybody,"\* perhaps it would be looked at with mounting hope. But after studying the text, we

would probably be disappointed to find that this is nothing more than a concentrated drill book, disguised by a careful distribution of photographs of airplanes, trains, skyscrapers, lathes, etc. Of course, the title might mean that *drill* is the universal use of mathematics. Or, perhaps a book will come bearing the title "Emergency Mathematics."\* We might think that this is something which surely goes to the heart of our war problem—but, again, we would probably be disappointed. For, here there might be many pictures and many problems and many formulas, but nowhere an attempt to get right down to fundamentals.

The most overworked titles in mathematics, during this war period, convey the idea that they offer something basic in mathematics—that here surely is the single pill dose of mathematics that is the panacea for all the world's mathematical ills. Unfortunately, the reader is doomed to disappointment, because the book that can do this has not been published. Many of the mathematical books in current vogue that bear titles intended to lead the reader into believing that these books are chock-full of "basic mathematics," "war mathematics," or "practical mathematics" are only designed for the mundane purpose of culling "war profits" from uninitiated readers. The disadvantages of such books are too great to enumerate without an inventory of the subject matter contained therein. We might briefly note these four general disadvantages:

1. these books take advantage of the *unschooled* readers' ignorance of mathematics;
2. the titles of these books are misleading in that the *teacher of mathematics* cannot be held responsible for teaching *war* applications;

\* This is, of course, a fictitious book.

3. these books do not make mathematics any *more* basic than it has always been;
4. the type of acquaintance that a reader of these boooks gains with mathematics is of *very doubtful value*.

These publications seem to extend and compound the "mythical" ills of a mathematical system that has been quite thoroughly subjected to the most careful kind of scrutiny. And the more we see of this, the more can we believe the seriousness of Lewis Carroll's Mock Turtle, who told Alice, "I studied Reeling and Writhng, of course, to begin with, and then the different branches of arithmetic: Ambition, Distraction, Uglification and Derision." The Reeling and Writhing are both outside of our sphere of influence, except the reeling and writhing occasioned by reading these books, but let us examine the four branches of arithmetic.

Somehow or other we can feel akin to Lewis Carroll when we look at present day mathematical literature. The ambition is there—the ambition of the author who attempts to guide wayward little mathematicians into green pastures of new applications when they hardly comprehend the elements of arithmetical processes involved. The distraction is present in the form to those totally useless exercises that serve no utilitarian purpose other than the mythical one for which the author designed them. The uglification arises from the distaste created for mathematics through the injudicious selection of material. The derision comes from that final dying gasp of the wounded student, who, in his most pained expression, asks "What's the use of doing it this way?"

All teachers should realize the shortcomings of their subjects. Unfortunately, too many teachers are prone to regard these shortcomings in much the same light as the weather—"everybody talks about it, but nobody does anything about it." Some teachers, on the other hand, do something about the shortcomings in mathematics—they select the best students and place

them in their own classes and leave the slower students to those teachers who are entitled to be called *teachers*. This does not meet the problem, but avoids it—if you can't teach them, scare them. Believing in a fundamental principle of democracy, the greatest good for the greatest number, a teacher must be prepared to teach all who come—the swift, the lame, and the halt—yes, and even the blind!

Some one may well raise the question now, if we are to teach mathematics to unselected students, what type of mathematics should we teach? Let's take an inventory of our stock in trade. Through years of experience, mankind has come to learn that mathematics is fundamental to the understanding of all knowledge. But it is not the whole body of mathematics as we see it in our textbooks that is so fundamental to other learnings, but rather that select part of mathematics that lends itself readily to adoption by philosopher, engineer, scientist, educator and theologian alike.

What parts of mathematics are these—what do they possess that other parts of mathematics do not possess? What makes them the basis of cultural democracy? Certainly they must be made up of the following:

Abstraction—one of the greatest strides in the direction of civilization came into being when man first realized that quantity need not be indissolubly associated with quality. Three dogs, three coins, three houses all have something in common—the abstract concept, *three*! Is this fundamental? Why? Because this type of abstraction enables us to apply the concept of number to all measurable qualities—to count. Is this fundamental to democracy? Certainly! Because it is highly desirable and without a doubt essential to recognize that in a democracy ten Negroes, ten Jews, ten Catholics, ten Protestants, ten Democrats, ten Republicans—all have the same essential abstract in common. Each of these categories make up ten citizens of our Democracy!

The Rational Processes of mathematics—addition, subtraction, multiplication and division—are they fundamental? Yes! The idea of combining the individual quantities making up the whole is fundamental to all learning—"the whole is greater than any of its parts"—what field of learning would not admit of this sweeping statement? Definitely basic to democracy, too. Why? Because our Democracy is made up of the various components: different "races," cultures, mores, languages, dialects, religions, nationalities—all welded into one complete whole—a whole greater than any of its parts; a whole for which men give their lives in battle that Democracy may be preserved!

Mathematics enhances the process of generalization and helps to form a clear picture of conditions under which the generalization may be valid or invalid. Such training is certainly of importance in a democracy where we wish our constituents to examine critically every dogmatic and non-democratic generalization pointed at the destruction of democracy. Accurate generalization is among the prime requisites of democratic life.

Mathematics aims to establish the idea of quantity changing through functional dependence upon some set of well defined variables. That is, to show the interdependence between functionally related quantities. Certainly this same type of thought is necessary to the unobstructed functioning of democracy—to bring out clearly how the exercise of some minority prejudices depend upon a non-democratic twist of thought processes.

These things, then, we recognize as basic: abstraction of numbers, counting, the rational processes of mathematics, generalization, and functional dependence. There is nothing in all of arithmetic and all the applications of mathematics to other fields of learning which is more basic than these. But we must not lose sight of one important factor. These things may only be considered basic on connection with their comprehension by the learner

and it may be necessary to employ circuitous methods to bring the learner to an appreciation of the basic concepts—although he may never reach the stage where they become basic in his own summary of this newly acquired knowledge. This would also seem to be the case in attempting to bring democracy to persons who have always been under fascist domination. The underlying philosophy that "all men are created free and equal" may only come to have meaning, to persons who have never known democracy, when they learn that democracy offers an opportunity to eat balanced meals, enjoy suitable clothing, earn higher wages, elect representatives to governmental offices, etc.

Hence, it should be unanimously agreed that in all our deliberations we must have in mind the pupils themselves (whether they be students of mathematics or residents of a fascist country), *with their interests*, varying as they do according to age and development. We must think as much about what *they like* to learn as about what *we wish* to teach them. It is of utmost importance that children should enjoy their work. If a subject is worth-while including in a curriculum, the pupils should not only have an opportunity to learn that subject, but should be given the opportunity to learn to love it. In the past, insufficient attention was paid to the sentiments and feelings which may grow up about a subject, yet these may form the very basis of permanent interest that should lead to real enrichment of life. This necessity for the pupils being interested in a subject makes itself evident, not only from the positive effect just mentioned, but lack of it undoubtedly leads to definite adverse sentiments and distaste.

It is agreed, nowadays, that interests gradually change as the child advances in years—just as democracy is a gradually changing awareness of the rights and needs of its constituents. It is found that young children are mostly interested when the facts presented are in themselves wonderful, startling, or extraordinary. Later they

pass from this "collecting stage" to a "utility stage," when they wish to study topics that are important in the world of practical affairs. They are interested in the practical activities of life as presented to them in the household, stores, streets and workshops; in the activities and occupations of the community at large. The young desire to enter into their life with the full activity of their senses, hands and thought, to gain added experience of it, to hear about it and to share in it, or, where they cannot actually share in it, to imitate it. (I think that is the message that progressive educators have been trying to get us to listen to.) That is, they eagerly seek participation—they want to belong and they want to contribute to this mental model of a democracy in action. But somehow or other, when they leave our school-rooms they leave this interest behind them much like the shedding of a snake's dead skin, and thereafter indulge only in apathetic forms of democracy.

Applications should be devoted to strengthening the understanding of basic concepts and extending them. Too many applications are such that the primary purpose is the application rather than the mathematics so basic to the solution of the problem. This makes mathematics *secondary* and the application *primary*. Every application must be so designed that the basic mathematical concept will not be lost in a useless welter of rhetorical details (which might very well project us headlong into the matter of *reading difficulties*). Many of the applications are so far afield from the domain of the mathematician that in many cases the teacher is placed in the awkward position of talking about something that he knows all too little about. Nonetheless, we should not oppose applications in the study of mathematics. That is, not applications that tend to strengthen and unify the internal bonds of mathematical learning. No more should we oppose the use of those democratic privileges that strengthen the bonds of our democracy—free speech, the right of as-

sembly, the ballot, the right to petition, etc.

There are some applications that are not basic, that do not develop understanding, that are used to taunt the student with *limited experiences* and elevate the ego of the student favored with *broad experiences*. These are the applications that create mistrust and breed misery, disgust and hate for mathematics. So, too, in a democracy may we not see abuses and misapplications of those democratic processes that we prize so dearly? How often have we seen free speech turned into a weapon for those who would weaken the very structure of democracy? Were these sound applications of democratic privileges? Are these the things we would favor over strengthening our foundations? We must evaluate our teaching in the light of its long range effect upon our democratic institutions!

Fortunately, school age children have a peculiarly righteous sense of judgment when it comes to applications of mathematics. They rebel against those applications which confound and confuse. And it is here that a teacher can be of the greatest service. Teach these children to apply basic criteria to their problems—little tests as to the purpose of the problems—do the problems achieve their purpose?—what is to be gained from mastery of the problems? Then learning must take the place of persuasion and compulsion—and it is these same tests which must teach an educated citizenry to discriminate between vandal, demagogue, fascist, and the liberal defender of democracy.

If students are not interested in mathematics, then what? Look at the material you have to teach—does it interest the pupils? Is it bound up with the lives of the pupils themselves in what they do and see in their homes, in the streets and shops, and what they hear about in the lives of other members of the community? Unless it is, it will not interest them. Most teachers would be willing to accept the material they teach upon the basis of this criterion. But look at it. To paraphrase the remarks

of one English writer,\* "We all go to the grocer's, sell pigs, buy coal, find the height of cliffs and the length of a guy rope on a ship's mast. A miserable bit of arithmetic, a little trigonometry, and Pythagoras, including far too many abstruse, inverse and complex problems on stocks and shares such as *rarely occur in real life*. Admitted too that modern textbooks give formulae taken from real life for exercise in logarithms and change of subject of a formula, but the latter is not needed anything as much in actual life as we are led to believe."

Look at the major portion of algebra and geometry. Can it be *real*? Have you ever heard of a farmer who forgot the price of sheep and cows and had to work backward from two bills he had? Have you ever known a mother who had to solve a problem based on the age of two other children to determine the age of her own child? What kind of ridiculous distortions of fact we must undergo in order to establish a few mathematical techniques! Has anyone ever seen a dimensionless point or a line that lacks thickness? How many senseless problems must we work before we can make sense out of nonsense? We might think of many senseless misuses of democracy, too—not the least of which would be an attempt on the part of a certain state to fix the value of  $\pi$  at 3. But these things need never exist. There are many things that are useful, that do touch on life, that are real, alive, interesting.

Is there anyone present today who has not thrilled at the applications of the Pythagorean theorem to something that he was constructing? Can anyone appreciate the full value of factoring unless they apply it to reduce the drudgery of their everyday computations? And what of the convenience of logarithms, determinants, Horner's method, etc., etc.? What of the deep cultural significance of mathematics? Can you picture our civilization without

the advantage of a number system? Can we picture a world devoid of mathematics? These are topics which are deserving of more serious thought. I think Hogben has a real point in Chapter I of his *Mathematics for the Million*. He is discussing the influence of mathematics upon civilization: "As mathematics is now being taught and expounded in our schools no effort is made to show its social heritage, its significance in our social lives, the immense dependence of civilized mankind upon it. . . . The first men who dwelt in cities were *talking* animals. The man of the machine age is a *calculating* animal. We live in a welter of figures—cooking recipes, railway time tables, employment aggregates, fines, taxes, war debts, overtime schedules, speed limits, bowling averages, betting odds, billiard scores, calories, babies' weights, clinical temperatures, hours of sunshine, motoring records, power indices, gas-meter readings, bank rates, freight rates, death rates, discount, interest, lotteries, wavelengths, tire pressures, . . . , ratio, limits, acceleration are not remote abstractions, dimly apprehended by the solitary genius, they are photographed on every page of our existence."

There is much to be done in teaching mathematics, and fortunately, a great deal of what has been done is good—quite good—even better than the critics lead us to believe. There should be no doubt of a job well done in our mathematics teaching if we have been sincere and conscientious and adaptable to the current trends in life. We might have overlooked some applications, but we would never have become aware of this if the overlooking hadn't made itself felt as a real need. And if some applications have been under-stressed, it is more the fault of those fields in which the mathematics should have been applied than it is the fault of the mathematics teachers.

We need have no fear that mathematics has not served its ends faithfully and democratically if we have kept our aims clearly in view at all times. Mathematics

\* Blackwell, A. "Some Doubts About Teaching Mathematics" in the *Mathematical Gazette*, Feb. 1940.

will fulfill even a greater social heritage than heretofore if we will be wise enough and bold enough to keep advancing with mathematics—to push back the barriers that oppose the spread of knowledge, that oppose the use of objectivity so inherent to mathematics, to fight for the advancement of one of democracy's greatest weapons—mathematics! And remember this, too, that Basic English may well turn to Basic Mathematics for aid because “one merit of mathematics no one can deny—it says more in fewer words than any other science in the world.”

Let us look forward hopefully to the time when we can all shoulder our democratic responsibilities of teaching mathematics for the greatest good to all, to make mathematics a vehicle of universal democracy, just as it is now the vehicle of universal truths, and to repudiate once and for all the stand expressed by Lewis Carroll in reference to the problems of society:

“And what mean all these mysteries  
to me

Whose life is full of indices and surds?

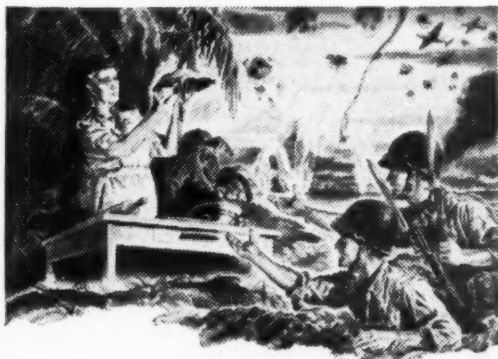
$$x^2 + 7x + 53 = 11/3."$$

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# Enriching Plane Geometry with Air Navigation

By HARRY SCHOR

Chief Ground Instructor, Polytechnic Institute of Brooklyn, New York and  
Instructor at the Manhattan High School of Aviation Trades

DEAD RECKONING leans heavily on the theorems and constructions of plane geometry. The connection between the two is, however, rarely emphasized. It is the opinion of this author that the teacher of plane geometry can use the problems of

$\vec{WS}$  is the velocity of the air relative to the ground, and  $GS$  is velocity of the airplane relative to the ground.

As in the case of all vectors, each of these has a magnitude and a direction listed below:

Vector:	Magnitude:	Direction:
$\vec{AS}$	Asp = airspeed of plane	TH = true heading
$\vec{WS}$	wsp = wind speed	WD = wind direction
$\vec{GS}$	gsp = groundspeed of plane	TC = true course

dead reckoning to enrich his course and help prepare his students for the air age. Many of the following suggestions can be used in the classroom, others in the Mathematics Club.

At the beginning of his course in Geometry the student is taught the concept of angle. Here the teacher can use bearings or courses as illustrations. Students should be encouraged to purchase 360° protractors. A bearing of 330° can teach the concept of a reflex angle; 160°, an obtuse angle.

Students often feel that a straight angle is not an angle at all. A bearing of 180° can make this concept clear.

Many texts in Geometry teach the basic constructions very early in the course. The wind triangle in its many forms can help in reviewing these concepts at the end of the course.

Fundamentally the wind triangle represents the fact that the velocity of the plane relative to the ground is vector sum of the velocity of the plane relative to the air plus the velocity of the air relative to the ground.

This may be represented by the vector sum

$$\vec{AS} + \vec{WS} = \vec{GS}$$

where  $\vec{AS}$  is the vector representing the velocity of the airplane relative to the air,

This makes a total of six elements in the wind triangle.

(True heading is the angle between the longitudinal axis of the plane and a line to the north pole; it is the direction in which the plane points.

True course is the angle between the intended flight path of the plane and a line to the north pole; it is the direction in which the plane is going.)

Now, the student of geometry is taught that three elements must be given to construct a triangle. One of these must be a side. In constructing the wind triangle four elements are needed, of the six possible elements, since one additional fact is needed to orient the triangle with respect to true north. The following examples will illustrate this:

Ex. 1. A pilot wishes to make good (1) a true course of 70° in a plane whose (2) airspeed is 90 mph when (3) the windspeed is 20 mph from (4) a direction of 330°.

Find: (5) the true heading required and (6) the groundspeed that will be made good.

This illustrates the construction known to the student of Geometry as "angle, side, side."

In the figure a line of indefinite length,  $OX$ , is drawn in a direction of 70°. This

orients the triangle. Then  $OW$  is drawn from  $330^\circ$  (toward  $150^\circ$ ). This is equivalent to constructing angle  $XOW$  which, is  $80^\circ$ . The length of  $OW$  is made 20 mph to a convenient scale. This is equivalent to constructing a side. Then with  $W$  as center

and the groundspeed of the plane.

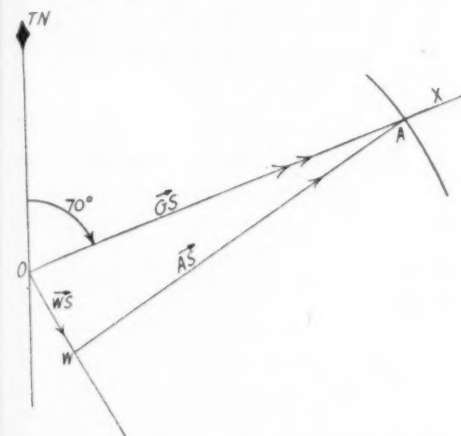
This problem illustrates the construction of a triangle by "angle, side, angle."

Many more examples of the use of the wind triangle can be found in air navigation but these should suffice to illustrate the point.

Not only constructions but also many theorems in Geometry can be illustrated by the use of dead reckoning.

For example, the theorem that if two parallel lines are cut by a transversal the corresponding angles are equal, explains why a rhumb line is a straight line on a Mercator Chart. (A rhumb line is a line which crosses each successive meridian at the same angle and the meridians are straight parallel lines on the Mercator Chart.)

The theorem that an exterior angle of a triangle is equal to the sum of the two remote interior angles can be used to explain why the angle that a straight line makes



and a radius of 90 mph to the same scale describe an arc intersecting  $OX$  in point  $A$ . This is the second side, and the triangle is constructed.

The length of  $OA$  is the groundspeed which will be made good. The direction of  $WA$  is the required heading of the plane. (answers: 92 mph and  $57^\circ$ )

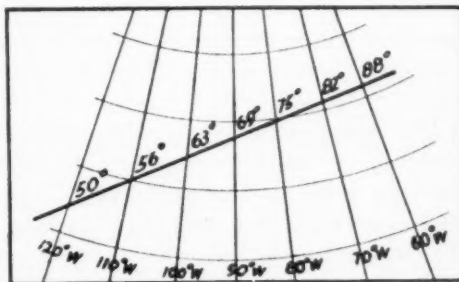
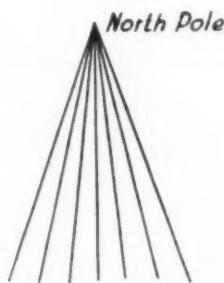
*Ex. 2. When a pilot (1) heads  $145^\circ$  at (2) an airspeed of 200 mph he (3) drifts  $8^\circ$  to the right and (4) makes good a ground speed of 220 mph. What is the wind speed and wind direction?*

This is equivalent to constructing a triangle given "side, angle, side."

*Ex. 3. A pilot keeps a heading of  $280^\circ$  at an airspeed of 140 mph when the wind is 30 mph from  $160^\circ$ . Find the course made good and the ground speed.*

Here too, the construction is "side, angle, side."

*Ex. 4. A pilot notices by smoke from chimneys that the wind is from the north. His drift meter shows that he is drifting  $10^\circ$  to the right of his heading. His compass when corrected, gives a true heading of  $90^\circ$  while his airspeed indicator when corrected gives an airspeed of 120 mph. Find the wind speed*



with successive meridians on a Lambert Conformal Conic Projection of the United States changes  $0.6305^\circ$  for every degree of longitude. On this projection the meridians are straight lines concurrent at the north pole. The angle at the pole between

two meridians one degree of longitude apart is  $0.6305^\circ$ . Therefore any straight line crossing the meridians forms a series of triangles as shown in the figure.

The parallel rules used in navigation can be shown to the class to illustrate the theorem that if the opposite sides of a quadrilateral are equal the figure is a parallelogram.

The interception problem can be used to illustrate the theorem that if a line is parallel to one side of a triangle it divides the other two sides proportionally.

3) Draw  $\vec{AW}$  = wind velocity.

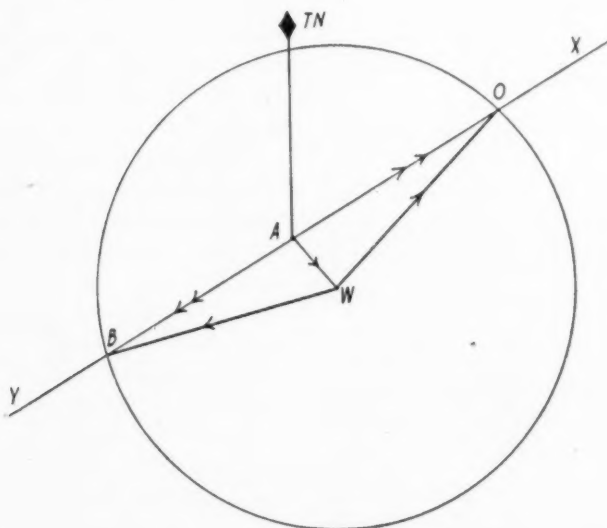
4) With  $W$  as center and radius equal to the wind speed describe a circle intersecting  $YX$  in  $O$  and  $B$ .

Then:

- a)  $AO$  is ground speed out. (111 mph)
- b)  $AB$  is ground speed back. (101 mph)

Prove:

- 1) Maximum distance pilot may venture out or, as it is commonly called, his



An interesting application of some of the theorems on circles can be found in the case of the simple radius of action problem.

*Illustration:* A pilot wishes to fly on a true course of  $60^\circ$  at an airspeed of 110 mph while the wind is 30 mph from  $320^\circ$ . He must return to his base at the end of one hour. How far out may he venture?

Construction:

- 1) Draw true north line.
- 2) Draw  $AX$  a line of indefinite length on a true course of  $60^\circ$  and extend to  $Y$  for the reciprocal course on the true course back of  $240^\circ$ .

Radius of Action =  $AO \times AB / AB$  (approximately 53 miles in this case).

2) Drift Angle  $AOW$  = Drift Angle  $ABW$  ( $16^\circ$  app.).

3) Maximum Radius of Action occurs when the wind is perpendicular to the course.

4) Minimum Radius of Action occurs when the wind is a headwind or a tailwind.

The first relation can be proved algebraically (see THE MATHEMATICS TEACHER, Feb. 1944, page 53), and the second by means of the theorem on the base angles of an isosceles triangle.

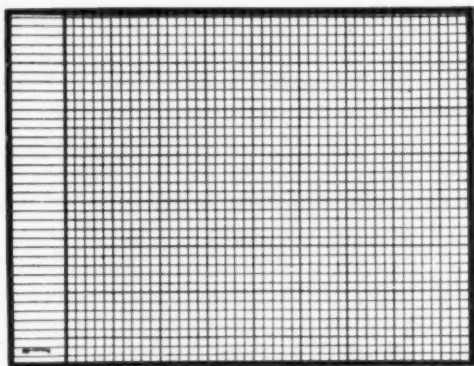
As for the last two, since the product of  $AO \times AB$  is constant, regardless of the

original direction of  $AO$  (product of the segments of all chords passing through a given point is a constant) then Radius of Action is maximum when the denominator  $OB$  is minimum (or perpendicular to  $AW$ ) and minimum when  $OB$  is maximum (or parallel to  $AW$ ).

These are but a few of the instances where Geometry can be vitalized by use of

air navigation. It would repay any teacher of mathematics to study the field of navigation thoroughly.

There are many other applications of locus, similar figures, and other topics of plane geometry used in air navigation but it is beyond the scope of this article to consider all these applications.



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# Developing the Principle of Continuity in the Teaching of Euclidean Geometry

By DANIEL B. LLOYD

Central High School, Washington, D. C.

THE DEVELOPMENT of modern mathematics has depended considerably upon the concept of functionality. Modern mathematicians have used it as the framework for much of their theory in the advanced fields of analysis and higher algebra. Descartes in the seventeenth century founded Cartesianism, an ingenious system whereby geometric loci can be represented algebraically. In this system, known as analytical geometry, any algebraic function corresponds to a geometric locus, and conversely. This algebraic representation of a curve or surface in space laid the ground work for higher geometry, —the analytical treatment of spacial elements. The paramount importance of functionality is thus evident in the entire field of modern mathematics.

In the teaching of secondary school mathematics the function concept as a unifying idea was stressed in 1923 by the National Committee on Mathematical Requirements in its famous report "The Reorganization of Mathematics in Secondary Education." The report favored shortening the list of geometric theorems to be proved, the breaking down of barriers between algebra, geometry, and arithmetic, and emphasis upon the function idea as a natural unifying concept.

Since the publication of the National Committee Report twenty years ago, an increasing amount of stress has been placed upon the function by teachers and textbook writers of algebra. By some it has been developed from the equation, or the formula, either separately or together; by still others it has served to introduce the equation, or the formula, or both. In surveying the existing practices it is difficult to discern which has predominantly been developed from the other, the function, the

equation, or the formula; it can be likened to the classical enigma; "Which came first, the chicken or the egg?"

But long before the function concept, came the *Principle of Continuity*, a principle recognized by ancient mathematicians. It probably dates as far back as the idea of generalization, with which it is associated. The principle of continuity is inherent in "locus" theory as developed by Greek geometers. It is implied in many theorems of which the following is an example: "All triangles on a common base, with vertex lying in a line parallel to said base, are equivalent." As applied to geometric elements it means that a certain relationship holds for all positions within certain locus limits. That is, a certain relationship is an invariant when elements or portions of the figure undergo a Euclidean transformation —that is, translation or rotation. Such invariants are concurrence of lines, collinearity of points, equality of areas of lines or angles, similarity, congruence, parallelism, perpendicularity, concentricity, etc.

The geometry of Euclid was never doubted to be the geometry of actual physical space until in the present century Einstein discovered certain minor discrepancies for bodies moving through great distances at high speed. Dedekind once stated: "The principle of continuity is obviously true because it agrees with everyone's representation or conception of a line." He pointed out the one-to-one correspondence between the infinite set of real numbers and the points on a line. Max Black\* said: "Intuition of the continuity of the visual field consists in apprehending (a) the connectivity of various portions of the field and (b) the possibility of infi-

\* Black, Max. *The Nature of Mathematics*, p. 87; Harcourt, Brace and Co., 1934.

nitely dividing any portion of it. The field is conceived to have no gaps, to hang together, and to be capable of division into successively smaller portions." Actually the retina of our eye has one discontinuity where the optic nerve enters, thus producing a blind spot and discontinuity in our visual field.

In modern times the principle of continuity has received more rigorous treatment in connection with Real Variable Theory, Complex Variable Theory, and Number Fields, such as the Dedekind cut in the continuum of real numbers. Modern function theory has thrown much light on continuity and developed the concept much farther than was formerly possible. The graphical representation of a function by using its related variables as coordinates in a suitable graphical system such as the Cartesian, reveals the continuity of the function, or the lack of continuity, in its locus. In higher geometry, continuity is given considerable attention in connection with the study of the properties of curves and surfaces.

In the typical textbook on elementary algebra the author may introduce the function concept inductively. He may present related pairs of numbers such as distance and time, for which, after inspection, the student is expected to set down a formula properly connecting the data functionally. The important feature here is this: The formula endows the functional relationship with continuity, a property which the tabular data can not express. Most physical data are capable of being continuous and thus the formula connecting them is a continuous function. Typical of these are most scientific and mensuration formulas. On the other hand, if the data are discrete they may still be related functionally, although the functions will be discontinuous. Example of these are the formulas for finding postage, freight, express charges, etc. Tables of related values would be adequate in place of formulas in the case of discrete data, provided range of the tables was sufficiently extensive.

The principle of continuity is well exemplified by trigonometric functions in the usual analytical treatment of the subject. Here we have the continuous functions, as the sine and cosine, and the discontinuous, as the tangent and secant. Just what happens at the critical points of discontinuity is well understood by studying their graphs.

In the light of the above status survey we as teachers of secondary school mathematics are led naturally to the next question: What is being done, or what should be done, about revealing the principle of continuity in Euclidean geometry? Here the author believes a definite weakness exists in current teaching practice and this he presents as his chief reason for writing this paper. He wishes to present a practical method for illuminating the continuity principle in high school geometry.

Every theorem proved involves a generalization, which is stated as the conclusion. The diagram though specific is so chosen as to be representative and admitting of a justified generalization. "Proved once, true for all" is our happy slogan and the students are particularly happy at the prospect of not having to repeat the torture. But do they "see" the generalization? Are they able to see beyond and visualize all the situations to which the proven fact applies? The ability of our teen-agers to generalize has been greatly over-rated in most cases. It involves greater maturity and experience than does the converse—namely applying the general to the specific. We should render more assistance and make more realistic this visualizing process. We need to implement or strengthen this transfer process or "carry over" to a variety of new situations.

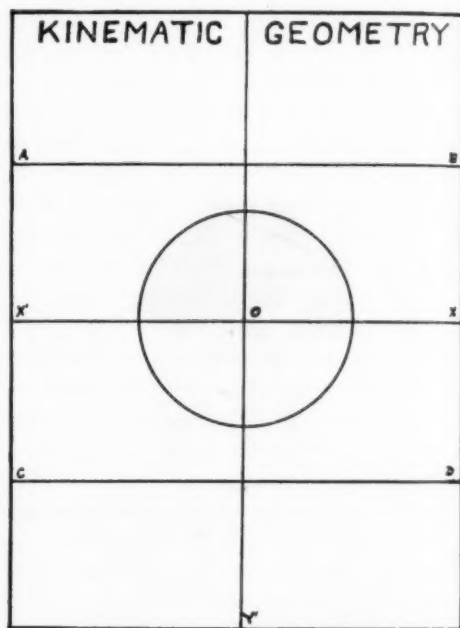
Traditionally, geometry has been studied in the tenth grade as a system of rigid, non-flexible elements; but the convention of using a static diagram is born of necessity and not of propriety. We can not analyze, or operate on, a figure that is wiggling around in the plane, and yet we must recognize that the facts we prove about

our chosen figure are equally applicable to each member of an infinite family of figures. We can vary the shape of the diagram without voiding the hypothesis or conclusion. The author believes that much insight can be gained by considering geometric diagrams as flexible linkages; and that correlatively with the usual proofs a study of the variance or invariance of properties of the diagram be made when any or all of the joints are flexed. It is not intended that the static diagram be eliminated in favor of the "kinematic" one, but the properties of the former can be elucidated by consideration of the latter, and incidental to the usual demonstrations.

The facts of Euclidean geometry become more interesting when viewed in the light of the principle of continuity. It is the unifying flux weaving the isolated threads into an intelligent matrix. Thus, we may observe that the perpendicular bisectors of the sides of a triangle may meet within, without or on a triangle, under various circumstances. Maybe we can draw a series of triangles and from them conjecture where the intersection will fall for the acute, right, and obtuse cases. Or maybe we can ascertain the facts by a series of proofs. (As a matter of fact, the student is not ready for these proofs at the time the circumcircle is studied.) My argument is here that a touch of realism can be added by flexing the triangle from the obtuse, through the right triangle position, and over into the acute form, observing meanwhile the corresponding shift of the circumcenter. This illustrates a typical case of geometric continuity.

A flexible diagram can be constructed on a cork bulletin board using elastic bands supported by thumb-tacks or push-pins. The author prefers a specially designed board which he has prepared called a "Kinematic Board" (see accompanying diagram). The word "Kinematic" is derived from the Greek word *Kinematōs*, meaning "motion." It has the same derivative as the well-known word "cinema," referring to "moving pictures." This board is

made of a piece of celotex, or similar wall-board, 3 feet wide and 4 feet high. A somewhat smaller size, as  $2\frac{1}{2}$  feet by  $3\frac{1}{2}$  feet, would be as suitable. It is rested on the chalk rail of the blackboard when used before a class. The lines placed on it are of red ink or red pencil—equally spaced horizontal lines, a central vertical axis and a circle with center at the origin.



KINEMATIC BOARD

The cost of the entire outfit is nominal, as a piece of scrap wall-board can be obtained at any lumber yard. The only additional equipment required are a dozen push-pins, the kind with large, long heads, and any kind of long rubber elastic bands, such as boys use for sling-shots, available at hobby shops. These can be looped at the ends, passed over the heads of the push-pins and thus held in place. As an alternative, if the long bands are not easily obtainable, ordinary rubber bands can be used, provided the figures constructed are smaller.

The author finds it helpful to use some of the small rubber bands to construct small figures at one side of the board. They can be set up in advance and can illustrate

a number of special cases while the main demonstration can be exhibited in the central portion of the board. The bands should not be stretched too much, as the pins might thus be dislodged. A little practice with this board will enable the demonstrator to put on a very striking exhibition of moving elements. It is entertaining as well as instructive, because it fascinates the class and motivates the work. The board is of such a size as to be easily stored when not in use, by shoving it behind or on top of a filing cabinet along the wall.

In practically every class a number of the students will be sufficiently interested to make smaller boards for their own individual use. They may vary from notebook size to larger. For these smaller boards, regular rubber bands and ordinary thumb-tacks usually suffice. These Kinematic Boards are helpful to students in visualizing relationships throughout their work in geometry.

As a visual aid the Kinematic Board is useful throughout the courses in plane and solid geometry. The diagrams that it can illustrate are so varied and numerous that it would be impractical to list them all here. The following list, however, is suggestive of its wide possibilities:

1. An inscribed angle in a circle remains constant as its vertex is moved around the arc.

2. An angle inscribed in a semicircle remains  $90^\circ$  as its vertex travels around the entire circumference of the circle.

3. An exterior angle  $C'$  of a triangle  $ABC$  equals the sum of the two opposite interior angles  $A$  and  $B$ ; but as  $A$  moves away from  $C'$  to infinity,  $\angle A \rightarrow 0$ ,  $AB$  and  $AC$  approach parallelism and  $\angle B \rightarrow \angle C'$ , thus verifying the equality of alternate-interior angles by the principle of continuity.

4. In problems with a polygon inscribed in a circle, to find arcs and angles; a vertex can be more readily moved by use of the Kinematic Board.

5. Start with an angle  $P$  between two diameters in a circle. Let  $P$  move from center to edge of circle, and thence without

the circle to infinity. Note how the arcs measure the angle for the several positions of vertex.

6. Exhibit the properties of various types of quadrilaterals and the relationship of their diagonals. Show a general quadrilateral with its diagonals; convert successively to a trapezoid, parallelogram, rhombus (or rectangle, square); with proper planning each conversion can be accomplished by moving only one vertex.

7. Illustrate the movement of the circumcenter and orthocenter of a triangle from its centroid to infinity as the triangle is converted gradually from an equilateral to an extremely obtuse type. Make the conversion by shift of a single vertex.

8. If two sides of a triangle are unequal, the angles opposite are unequal, and conversely. Convert an isosceles type to various types of scalene triangles.

9. The area of a triangle remains invariant as its vertex moves on a line parallel to its base.

10. Construct a triangle by wrapping two or more rubber bands around three pins (instead of just one band as is usually done). Successively increase the number of sides of the polygon from three, by taking pins and pulling the bands, one by one, outwards. This builds a polygon of any desired number of sides and the sum of its angles may be studied in a new manner.

The board is particularly helpful in locus problems in which the ubiquitous property of continuity is essential. Also in solid geometry it is useful, as for instance, in studying the properties of a polyhedral angle when its vertex is moved away from the plane of its base. The resulting variation and limiting values of the sum of its face angles and dihedral angles become thus much clearer.

Other flexible linkages for illustrating geometric relationships can be devised besides the Kinematic Board and students should be encouraged to originate and construct them. Thus, a parallelogram with hinged joints can be constructed of wooden strips and equipped with elastic

diagonals. Again, two wooden bars or sticks connected by several parallel elastic bands would illustrate theorems about parallels cut by two transversals.

The principle of the Kinematic Board enables one to pass from one geometric situation to another through a continuous series of positions. The Kinematic diagram bears the same relation to a series of static diagrams as a continuous series does to a discrete series of values. The flexible

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# Problems of Teaching Mathematics in the Schools of England\*

By A. HOOPER

IT WAS with mixed feelings of surprise, and may I say, of trepidation, that I received an invitation from our Chairman Mr. Melby, to come and address you this afternoon. What qualifications had I that would justify so great an honor as that of addressing a gathering of this nature? Much as I hate to disillusion all those who have ensured a prompt reply to their letters by addressing me as "Professor" I have to admit that I can lay claim to no such lofty title. I am just one of the rank and file, who has spent more than twenty-five years in the classrooms of English Secondary, or High Schools and English private schools. No, I am just a plain, ordinary teacher.

But since I understand that the majority of those present this afternoon are, like me, plain, ordinary teachers (though looking round me I feel I must withdraw the adjective "plain"). I feel sure that you, at any rate, will agree with me when I say that we are the people who really matter, we are the front-line troops who really count when it comes to tackling one of the greatest—if not the greatest of problems which confront any nation, the problem of the education of the men and women of tomorrow, in whose hands will lie the destiny the welfare and the happiness of the whole nation. More than that, I venture to suggest that today a further heavy responsibility falls on every English-speaking teacher throughout the world, a further inspiring opportunity opens before every such teacher, no matter what his or her subject may be, the responsibility and the opportunity of impressing on those entrusted to their care the fact that on the nature of the characters and the measure

of the intellects of English-speaking men and women will rest, to a very great extent the welfare and happiness of most of the men and women on this Earth.

It is an obvious truism to say that the teaching profession should be regarded everywhere as the highest, the most exacting, the most difficult-of-entry and the best paid of all the professions. Possibly, in America it is all these things. It would be an impertinence on my part to discuss such matters. But I can assure you that so far as England is concerned, few, if any of these attributes can be associated with the profession to which is entrusted not only the moulding of the characters and minds of the men and women of tomorrow, but also the laying of the foundation on which alone the structure of all the other professions can be based. And therein lies one of the greatest and most pressing of the problems which face our profession (I speak, of course, for England), the urgent necessity for bringing home to others the immense importance of our profession and the dignity and attractiveness which should be associated with it. In Roman days, teachers were slaves—mere domestic chattels, and still today, many people who ought to know better are inclined to look down their noses at our profession. I think it was G. B. Shaw who, in a typically Shavian utterance, said "He who can, does; he who can't, teaches." Unjust and unmerited though such a snap-judgment is, it yet undoubtedly indicates the attitude of a large number of people even today, and it is this failure to recognize the supreme importance of teaching and teachers that prevents sufficiently large numbers of men and women of outstanding ability, achievement and ambition from entering a profession which socially and financially is still, for some reason, regarded as infe-

\* An address delivered before the Mathematics Section of the Wisconsin Education Association on November 4, 1943.

rior to other professions. Until we raise the level of teaching to at least that of the legal and medical professions we shall fail to attract such men and women in sufficiently large numbers as will make possible a tightening up of the qualifications prerequisite to entry and a consequent raising of the level of education generally. What I have said applies more especially to the teaching of mathematics. In normal times it is comparatively easy to secure an adequate supply of teachers of English and History and languages, but the mathematician can as a rule sell his brains to so much greater advantage in other walks of life that it is extremely difficult to secure the services of a really first-class mathematical teacher. Of course, there are many idealists who enter our profession. I don't know where we should be without them—and I know that you are fortunate over here in having your full share of such idealists, for whom material gains count for little or nothing. But I say that idealism is not enough. The first thing we must do is to convince those responsible for financing education that the finest investment they can make is to provide the finest possible education for the men and women of tomorrow. During the early days of the war I was stationed in the Orkneys, far away to the north of Scotland. And to my surprise I found, in a little village called Finstown, a most delightful schoolmaster, a graduate not only of one of the great Scottish Universities, but also of Heidelberg, and a man of great culture and influence, not only among the young people but also among the whole community. And he told me that there was little difficulty in securing well-qualified teachers right up there in those remote islands, for I gathered, although the Orcadians have the reputation of being hard-headed men of business,—they are reputed to have the largest individual bank balances of any part of Scotland, which is saying something,—they recognize that education is an investment which pays the most certain and the most handsome of dividends and they see to it that their

teachers are remunerated equally as well as their doctors and lawyers. And I believe I am right in saying that as a consequence there are at the present time no fewer than seven professors at great British universities who received their primary education in those little schools in Orkney villages.

Unfortunately, this happy state of affairs does not apply to England as a whole, hence arises one of our great problems, the recruitment of sufficiently large numbers of the finest type of teachers. This being so, it is perhaps fortunate that we are not faced with quite the same problems as, I understand, confront you in America. To begin with, England is a tightly packed little island (I speak of England and Wales) with nearly forty million people crammed into an area little larger than the State of Wisconsin. The majority of our people are either town dwellers or live so close to town that it is a simple matter, in normal times, to centralize the Secondary Schools (State High Schools), and also, to a very large extent, the elementary schools. Thus, we are not faced to any appreciable extent with your problem of staffing a tiny school with perhaps twenty and thirty pupils of all ages and grades. As I pass through your villages and see your little red brick schoolhouses in which so many of your great men have received their primary education, I am filled with admiration, if not with envy, for the teacher who, single-handed, tackles the education of children at eight different grades and such diversity of ages.

Then again, I imagine the task of the English teacher is made easier by the fact that entrance to a Secondary or Public School (as we, being English, call our great Private Schools) is selective. This does not mean that we in England do not believe as strongly as you in the right of every boy and girl to enjoy equality of opportunity. Far from it: before the war, more than half the under-graduates at our universities received aid from public funds, some of them being entirely supported

by scholarships and grants from public sources, and today, any boy or girl of ability and ambition can work his or her way from the elementary school right through the university. But we consider it is a waste of time, effort and money to attempt to give this sort of education to every boy and girl, regardless of their ability to benefit from it. This means that our teachers in High Schools at any rate find they have a reasonable standard of intelligence in their classes. Moreover, promotion from form to form (i.e., from grade to grade) is by merit and is not automatic, consequently a teacher finds that the majority of those in his class are at the same approximate level of mental development, which is a great aid. Even so, most of our teachers in State Secondary Schools feel that their classes are too large, though they are not nearly as large as they were when I taught in one more than twenty years ago. It is felt that in all our schools, the limit to the number of pupils in a class should be thirty, though this aim has not yet been fully achieved.

I have mentioned certain advantages which, it appears to me, are held by English teachers. On the other hand, I imagine that your High School teacher of mathematics has an advantage in the fact that all those in his mathematics classes have elected to take the subject. In England, mathematics is compulsory until a pupil has passed the School Certificate examination. This is an examination conducted by external examiners, the passing of which is regarded as evidence of a sound general education. Up to eight subjects may be taken, of which certain subjects, including mathematics, are compulsory. If a sufficiently high percentage is obtained in certain stated subjects, exemption is granted from the entrance examination to any of the British universities, while most of the professions accept such School Certificates in lieu of their preliminary examination. The fact that the vast majority of our High School mathematical pupils are thus conscripts and not volunteers

makes the task of the teacher more difficult. His reaction may be either to strain every nerve and spare no effort in developing the interest of his pupils, or he may slide into a groove and plough through the same old exercises in a deadly, mechanical way, never caring whether he arouses a spark of interest in the subject; whether his pupils catch the least glimpse of the beauty and fascination of mathematics or any inkling of the essential part played in modern life; or whether they leave his classes imagining it a loathsome subject consisting of the memorization and application of an endless, meaningless and disconnected string of mechanical rules.

It must not be supposed that every pupil in an English Secondary School passes the School Certificate examination, or even reaches the form, or Grade, in which the examination is taken. Far from it. And therein lay one of the great problems with which we were faced when, in 1939, we were forced into a war for which we had no more desire than you had, and a war for which we had made no greater preparation than had you. We had to supply large numbers of men capable of applying at least simple mathematics, and we had to supply them in the shortest possible time. Our very existence was at stake, and every moment counted.

So far as the R.A.F. was concerned, before the war, it had been possible to hand-pick its members. When, after the last war, that wave of idealism and pacifism swept over England as it did over America we proceeded to cut down our once supreme air force. Fortunately, the then Secretary of State for Air insisted that though his force must be curtailed as to numbers, he would have the finest equipment procurable and the finest type of personnel. (I can say this, for I did not become a member of the Air Force until after the war had started, by which time the standards had been relaxed!). I venture to think that it is a good thing for all freedom-loving men and women throughout the world that such high standard was

set. As, however, the R.A.F. expanded with amazing rapidity, it became necessary to lower the very high standards which had been set up, so far as educational requirements for entry were concerned (though the final requirements today are as searching as ever) and then it was that many of us who had been teaching mathematics all our lives received a severe, yet, I venture to say, salutary shock. Those of us who were honest with ourselves had to admit that there had been something radically wrong with our method of teaching the subject. It was very good, so far as those with a natural aptitude and liking for mathematics were concerned, but we found there were large sections of our former pupils whose knowledge of mathematical processes was chaotic, fragmentary and vague in the extreme. Moreover, many of them had conceived a dislike bordering on hatred for the subject and had developed a kind of mathematical inferiority complex.

I venture to think that events have proved that the vast majority of them were cured in an amazingly short time. These are the men who now, night after night, pilot great bombers in pitch darkness without benefit of wireless-telegraphy-direction-finding over far-distant territories of Europe, and, except for the average five per cent shot down, bring them safely home to their bases. These men by their achievements clearly demonstrate one fact, a fact which I think is the crux of all successful mathematical teaching, *that once a person has a sufficiently strong desire and motive for learning the subject, he can master it with ease.* I spent nine months at Scapa Flow, in the Orkneys, with a branch of the R.A.F. sometimes alluded to in R.A.F. slang as "Balloonists." These men were responsible for flying those great barrage balloons which object is to force the enemy high up above his target and into the range of the ack-ack guns. I know of no more trying and boring existence than that of these men, anxious to do their part in

the struggle for existence and freedom, yet forced to remain—as they incorrectly imagined—passive spectators. Many of them were on lonely and desolate islands—we were delighted to find a local pre-war guide book which described some of the islands on which we were stationed as "uninhabited and uninhabitable"—we had no option but to disprove the latter adjective! These men were prepared to go even to the length of learning mathematics in order to qualify as more active participants in the struggle. I have seldom known greater satisfaction than I had from teaching such men. *They had a motive*; they realized for the first time that mathematics plays a vital part in modern life; they soon found that their fears and inhibitions were no more substantial than were those mists which so often shroud those islands in that northern sea.

Then I was transferred to an R.A.F. Air Navigation school in Canada and there I came in contact with many of your American boys who joined the R.C.A.F. before you came into the war. Those who had taken mathematics in your High Schools had no difficulty with the subject, though many of them were naturally rusty. Those who had not taken mathematics in High School, or had not been to High School were not so good. But the point is this: the vast majority of them made good, and made good in a very short time. And the reason was, that once again, *they had an interest and a motive in learning the subject.*

Here we have the essential problem which faces every teacher of mathematics: the problem of motivation and of arousing the interest of his pupils. Too many of us have in the past been content to allow our pupils to acquire the mistaken notion that the subject is a mystery to be understood only by those born with a queer sort of mental kink.

At this stage I am going to be very daring: I trust you will not feel I am being presumptuous. At the suggestion of one of your High School teachers I am going to

offer for your consideration an introductory talk which I believe is useful in arousing the interest of our pupils. I want you to imagine that you are pupils beginning your High School career and that this is your first mathematics class. I am very conscious that I am addressing many experienced teachers who have their own successful technique, and for whom what I have to say can have no interest or value. I ask them to bear with me and to adopt the philosophical attitude of the Thibetan whom Colonel Younghusband is reported to have met sitting by the roadside and hitting himself on the head with a hammer. When asked "Why do you do this? Doesn't it hurt?" he replied "Of course it hurts, but think how nice it will be when I leave off."

Now our pupils come to us with many varied interests, likes and dislikes, but all of them like pictures. You may remember how the poet Chaucer in 1391 wrote a treatise on the astrolabe for his son "Litel Lowis" whom he had sent to Oxford University at the age of ten, and who apparently was unfortunate enough to be compelled to study what we should now call Spherical Trigonometry at that age. Whenever Chaucer felt he had completely muddled his Litel Lowis by his verbal descriptions, he would draw a picture and add the words "and for the more declaracion, lo here thy figure."

So I start off by showing my pupils a picture of the River Mathematics. Here it is. You will see that on the left is a timescale. Below the 2000 B.C. line all the dates are uncertain and conjectural, but above that line the approximate extent of mathematical knowledge is indicated by the breadth of the river. You will notice a great sand-bank or barrier which splits the river into two branches until about the year 1500 A.D. One branch is labelled *Number Reckoning* and you will see that it does not appreciably broaden out until a tributary marked *Hindu-Arabic number symbols* flows into it about the year 800 A.D. There is indeed a little tributary in 250 B.C.

labelled *ARCHIMEDES* but this brings about little extension of the main river. (At this stage, I should deal briefly with Roman and "Arabic" Numerals and the abacus, since all our manipulation of numbers is based on the idea behind that abacus). You will see that the other branch of the river broadens out suddenly when it receives tributaries marked *Thales*, *Pythagoras*, *Plato* and *Euclid* but then remains more or less static until the great tributary marked *Astronomical Measurement of angles, chords etc.* flows into it about the year 1500 A.D. Meantime, on the other side we see the junction with the main river of rivers marked *Al-jabr* and *Abstract Science of Numbers* (I have refrained from calling this *Aritamētikē* owing to the confusing change in significance in the word). You will see that it was not until comparatively recent times that the great barrier was finally overcome and the waters of both sides, enriched by the waters of the various tributaries enabled to mingle together in the form of *Trigonometry*, *Analytic Geometry* and the *Calculus*. On these broad waters we see three ships sailing, which we may liken to the *Pinta*, *Nina* and *Santa Maria*, for their names are *Science*, *Engineering* and *Aeronautic* the modern ships of discovery whose never-ending voyages into the unknown offer to mankind the possibility of a new world of freedom from want, and consequently, of freedom from fear. Without the River Mathematics these ships could never have been launched; only its never-failing flood keeps them afloat—without it they would be stranded high and dry, their mission unfulfilled, their treasures lost.

Time does not permit me to enlarge on the many fascinating details that can be pointed out by a teacher who knows his subject and himself feels its fascination; on the way in which a pupil's interest and curiosity can be aroused and an appreciation of the part played by mathematics in modern life implanted in his mind as well as coordination of the various branches

of the subject. I feel that I have already hammered your heads sufficiently, but before I sit down may I express my deep appreciation of your action in inviting me to speak to you today. It is, I feel, a happy augury for the future that English-speaking members of the teaching profession should get together whenever possible and mutually confirm their faith in the part they are called upon to play in laying the foundation of a better world. When Mr. Churchill was addressing the members of your Congress and Senate he used these

words "If I may use other language, I would say that he must indeed be blind of soul who cannot see that some great purpose and design is being worked out here below, of which we have the honor to be the faithful servants." If you believe, as I believe, that amid all the tragedy and suffering some great purpose and design is being worked out here on this Earth today you will agree that we teachers of the English-speaking peoples of the Earth must play a real and vital part in the fulfilment of that purpose and design.



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# A Check List for Pre-Induction Mathematics

By VIRGIL S. MALLORY

*State Teachers College, Montclair, N. J.*

IN THE October 1943 issue of THE MATHEMATICS TEACHER a committee<sup>1</sup> of the National Council of Teachers of Mathematics, working with the cooperation of the Civilian Pre-Induction Training Branch of the Army Service Forces and the U. S. Office of Education made a report on Essential Mathematics for Minimum Army Needs.<sup>2</sup> The purposes of the report were to emphasize the needs of the inductee for certain minimum essentials in mathematics, to present a list of the essentials needed, to recommend placement in the high school curriculum of those essentials not already there, and to give some guidance for teaching them. Incidentally the report pointed out the value in civilian life of these same essentials.

The minimum essentials of mathematics listed in that report were determined by visits of the committee members to Army camps and by conferences with over three hundred Army officers.

The report suggested how, without the establishment of new courses in mathematics, these minimum essentials could be incorporated into the regular courses in sequential mathematics or in general mathematics. It will be noted that major emphasis was given to those more elementary aspects of mathematics which are used by practically every inductee. As the report stated, despite the fact that these are simple elements of mathematics, it should not be assumed without careful checking that they have been mastered by pupils. Ample evidence shows that the typical inductee does not have competence in their use.

<sup>1</sup> The committee consisted of Rolland R. Smith, C. Louis Thiele, F. Lynwood Wren, Wm. A. Brownell, John Lund, Giles M. Ruch, and Virgil S. Mallory, chairman.

<sup>2</sup> Reprints of that report may be secured from William D. Reeve, Teachers College, Columbia University, New York City, for 10¢.

A year has passed since the publishing of the report and it seems pertinent to provide teachers of mathematics with a check list so that they may determine the effectiveness of their teaching of this type of mathematics. Such a check will be of great help to those boys who are preparing to enter the army as well as to boys and girls planning to engage in industry.

The cooperation of every high school teacher of mathematics is invited in the use of this check list.

## CHECK LIST

1. Can the pupil read and write with proficiency and understanding
  - a. Whole numbers to eight digits?
  - b. Common fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, 16, 32, and 64?
  - c. Decimal fractions to ten-thousandths?
  - d. Per cents?

These are used in serial numbers on rifles and identification tags; sizes of wrenches, bolts, and nuts; frequencies in kilocycles, valve clearances; per cent of men assigned to details and proportions of ingredients in mixtures.

2. Can the pupil count by 1's, 2's, 5's, and 10's with accuracy the number of objects in a group?

This ability is used in counting the number of men, automotive units, or equipment; in counting paces for determining distances.

3. Can the pupil perform the fundamental operations on whole numbers with understanding, accuracy, and reasonable speed?

These are used in stock, mess, and supply reports; amount of ammunition and materials consumed; in reading map grids; in determining mileage; in radio frequencies; in changing units of measure.

4. Does the pupil understand funda-

mental operations with common fractions and thus have skill in their use?

These operations are used in reading blue prints, calculating seam allowances; in tap and dye sizes, depth of cut in machine work; in determining amounts of ingredients to be used in mixtures; finding weights of castings; in electrical problems, in making switchboard installations.

5. Does the pupil have a clear understanding of the system of decimal notation basic to the use of decimal fractions?

6. Has the pupil such a concept of decimal fractions that he can display it by: determining the relative size of two decimal fractions; give the result of multiplying or dividing a decimal fraction by a power of 10; correctly place the decimal point in a result by inspection?

7. Can the pupil add, subtract, multiply, and divide decimal fractions with understanding, assurance, and skill?

This work with decimal fractions is used: by supply clerks in finding the total money value of rations; in checking air craft specifications and in machine shop work; in finding frequencies in electrical work; in laying out rivet patterns and in determining costs.

8. Does the pupil have so clear an understanding of part-whole relationships with common and decimal fractions and per cents that he can

a. Find a required part of a number when the part to be found is expressed as a common or decimal fraction or as a per cent?

b. Find what part one number is of another and express the result as a common or decimal fraction or as a per cent?

c. Find a number given a part and its relative size as a common or decimal fraction or as a per cent?

These are used in computing dial settings; in finding speeds for different sized gears or pulleys, determining slopes in grade stakes; in finding amount of product in Army baking operations.

9. Does the pupil have so clear a concept of the ratio concept expressed as a

common or decimal fraction or as a per cent that he can solve practical problems in ratio?

Ratio is used in concrete and fuel mixtures, in scales in aerial photographs and maps, in slopes and grades, in speeds and sizes of pulleys and gears.

10. a. Does the pupil understand the symbolism for powers and can he find powers of numbers?

b. Can he find square roots from tables?

These skills are used in the common formulas in many branches of the Army, in finding a side of a right triangle or the diagonal of a rectangle. The understanding of powers (particularly of powers of 10) and of the square roots of even powers of 10 is needed for adequate use of a table of square roots.

11. Has the pupil had such thorough training in the use of graphs, maps, and grids that he can

a. Understand such representative fractions (R.F.) as are used in military maps as 1/20,000, 1/50,000, etc.?

b. Interpret maps and grids and find directions and distances from point to point?

c. Use maps, grids, and graphs intelligently?

d. Make maps and graphs?

These skills are used in determining direction of travel to reach an objective; to orient one's own position; to discover location of artillery, command posts and hostile forces; for scouting and reconnaissance, for radio and signal communication.

12. Has the pupil had adequate practice so that he is skillful in using tables?

This ability is used in finding angle of elevation for an artillery piece from firing tables, making corrections for windage, finding strength of materials, fractional equivalents, squares and square roots.

13. a. Does the pupil understand the symbolism of algebra, how to use letters to represent numbers?

b. Has he memorized simple formulas for areas and volumes, and the formula relating to distance, rate, and time?

c. Can he substitute numerical values in simple formulas to obtain a value for one of the letters?

d. Can he solve simple algebraic equations?

e. Does he understand the meaning of positive and negative numbers and can he use them intelligently?

These skills in algebraic technique are widely used in many branches of the services. Signed numbers are used in electrical work, in machine gun firing tables, and range tables, in indicating direction and time.

14. Has the pupil had practical as well as theoretical training in measurement so that he

a. Understands the basic concepts of weight, area, and volume and can perform the fundamental operations with denominate numbers?

b. Understands the measurement and the units of measurement for temperature (C and F), angles, and time?

c. Knows the more common units of the metric system, can use them in measurements and knows their simple equivalents to English units?

d. Has had practice in using such measuring units as the tape, rule, compass and protractor; magnetic compass; transit or use of protractor for outdoor measurements; calipers and dividers; steel square and level; scales?

e. Has had practice in estimating numbers, weights, heights, distances, and speeds?

f. Understands limits of accuracy and tolerances?

These uses of measurement have wide application in every branch of the Army. Some of these are: determining storage tank capacity, area of fire, load limits, finding tension for a wire, machine shop work, blue print and map reading; in medical measurements, radio repair, and designating gun calibre; in determining directions, road building and communica-

tions constructions; in carpentry, and engineering.

15. Has the pupil had enough training in geometry, particularly informal geometry, so that he has correct concepts of

a. Point; straight, curved, horizontal, vertical, oblique, parallel, and perpendicular lines; angle; and slope?

b. Triangle (right, scalene, isosceles, and equilateral), parallelogram (square and rectangle), trapezoid, circle, ellipse, and regular polygon; prism, cylinder, cone, and sphere?

c. Similarity?

These are used in the technique of fire, map reading, and the grade of a road; in cone of fire, aiming circle, and machine cams; in pattern making and automotive parts.

16. Has the pupil had enough practice in drawing and construction so that

a. He can use with skill a ruler graduated in 32nds and in 10ths of an inch, and in millimeters; compasses to construct circles and protractors to measure and to draw angles?

b. He can construct a perpendicular to a line?

c. He understands views of a simple object as given in a scale drawing, blue print, or sketch?

d. He can use the 3-4-5 relationship in a right triangle?

These abilities are used in preparation of parts for welding, radio repair, laying out range charts, determining azimuths; in the construction of bridges, machine shop work, the thrust-line system of establishing a rendezvous; in fabrication of materials, functioning of rifle, blue print reading; in carpentry and the sighting of mortars.

17. Do pupils understand the meaning of and are they skillful in using rounded numbers; the mean, median, and mode?

These are used in calculation of rations, marksmanship scores, constructing a pace scale; in Medical Corps statistics.

# ◆ THE ART OF TEACHING ◆

## The Use of Models in the Teaching of Plane Geometry

By FRANCES M. BURNS

*Oneida High School, Oneida, New York*

THE USE of models in the teaching of solid geometry has long been an accepted practice, but in plane geometry they have found little favor. Although not complicated by a third dimension, many of the relationships of plane geometry are difficult for beginning students to understand. The visual impression created by a model often clarifies the meaning of a proposition or leads to a generalization. The purpose of this article is not to present a case to justify their use, but rather to indicate some ways in which the writer's plane geometry classes have found them helpful.

An essential feature of most of the models is that they consist of *movable* parts by means of elastic, dowels, hinges or nuts and bolts. With a few exceptions they have been made by the pupils.

Some ways in which the models have been used are:

### *I. To determine the probable conclusion which follows from given data.*

When a pupil proves a theorem which has been stated completely, he is getting practice on making a proof but is not necessarily developing an ability to discover relationships between parts of a figure. The construction of many of the models is such that the controlled parts represent the hypothesis, while that which follows from this hypothesis is indicated by elastic or other movable parts. Various positions of the controlled parts can be shown in a few

seconds and the conclusion observed not for one position of the figure, but for several. On the models the pupil can easily see which parts are controlled and which parts follow as a result of this, whereas on a black-board diagram this is often not clear to him. Hypothesis and conclusion thus become more meaningful. Some of the theorems developed in this manner are:

1. Facts about the diagonals of the various parallelograms. Plate I-10 and Plate IV-14.
2. Two points each equidistant from the ends of a line determine its perpendicular bisector. Plate I-11.
3. All the angle measurement theorems. Plate I-19 and Plate III-1, 3, 4, 6.
4. The line joining the midpoints of two sides of a triangle is parallel to the third side and equals one half of it. Plate II-6 and Plate IV-20.
5. If a line divides two sides of a triangle proportionally it is parallel to the third side. This model will also show the inverse. Plate II-5.
6. The exterior angle of a triangle is greater than a non-adjacent interior angle. Plate II-9.
7. If a diameter is perpendicular to a chord it bisects the chord and its arcs. The converse theorems can also be shown. Plate IV-5.

8. Changes in the trigonometric ratios for an acute angle. Plate IV-17.
9. Other relationships suggested by Plate II-4, 7, 8, 11, 12 and Plate III-2.

The pupil realizes that the use of a model does not constitute a proof and that a conclusion thus drawn is inductive. Usually a relationship is proved deductively immediately following the use of the model.

## II. *Instruments whose construction or whose use employs simple geometric principles.*

1. The parallel ruler. If the opposite sides of a quadrilateral are made equal, then it is a parallelogram. The one shown in Plate IV-18 is large enough for blackboard work.
2. Proportional dividers. If two triangles have an angle of one equal to an angle of the other, and if the sides forming these angles are proportional, then the triangles are similar. Corresponding sides of similar triangles are proportional. Plate IV-15.
3. Angle mirror. The locus of the vertices of all right triangles with a given hypotenuse is a circle whose diameter is that hypotenuse.
4. Center square. Tangents to a circle from an external point are equal. The bisector of the vertex angle of an isosceles triangle is the perpendicular bisector of the base. The perpendicular bisector of a chord passes through the center of the circle. Plate IV-16.
5. Tool for drawing similar triangles at the board. Plate IV-4.
6. Carpenter's square, hypsometer, pantograph.

## III. *To correct wrong responses.*

1. Plate IV-13 is used when a pupil says, "Two triangles are congru-

ent if two sides and any angle of one are equal respectively to two sides and any angle of the other." Plate II-1, the ambiguous case of the law sines, can also be used here.

2. Plates I-10 or IV-18 are brought out when the class thinks that parallelograms with equal perimeters have equal areas. It is then easy for them to see that the area changes with a change in the altitude and that, of these models the rectangle has the greatest area.
3. Plate IV-7. In the first few weeks of geometry a class will correctly solve the equation  $x - 2 = 10$ . When asked *how* it was solved nearly all will say, "You transpose the 2 to the other side and change its sign." If you pretend not to understand and ask a pupil to come up and carry the  $\boxed{-2}$  over to the other side of the equals sign, getting  $x = 10 - 2$ , the class sees that there is no magic by which the minus sign changes to a plus sign. (The minus sign must be printed on the block with the 2 as  $\boxed{-2}$ . Otherwise the pupil will get  $\boxed{x} = \boxed{10} \boxed{2}$  and will say, "If there is no sign before the 2 it means plus and so the sign got changed coming across.") Strictly speaking this may not be an error, but it gives the opportunity of emphasizing the real method of solution for certain linear equations and of illustrating a use of axioms.
4. If a poor pupil says, "If two triangles have two sides of one equal to two sides of the other, then their third sides are equal," Plate IV-19 is used.

## IV. *To emphasize the difference between drawing conclusions inductively and deductively.*

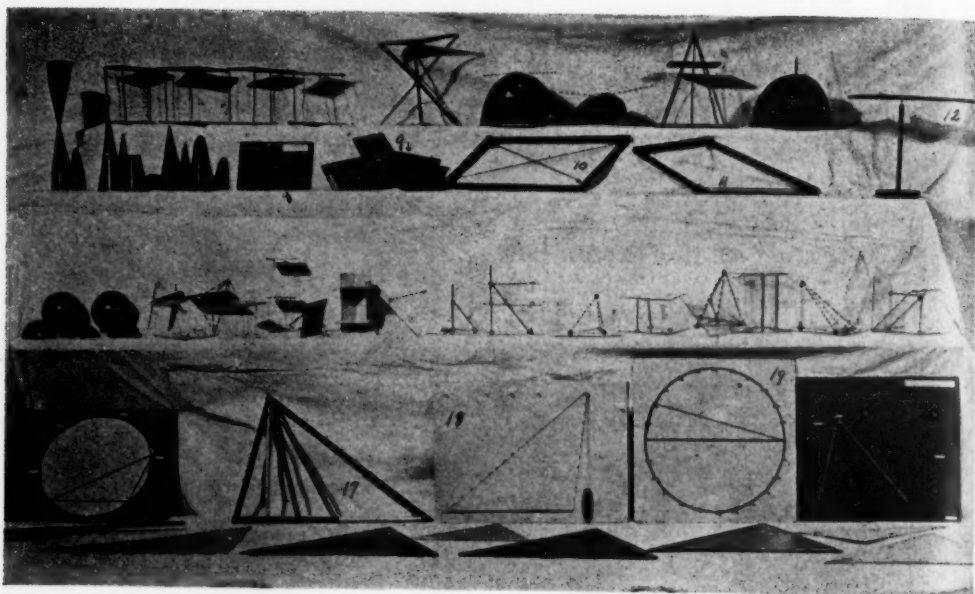


PLATE I

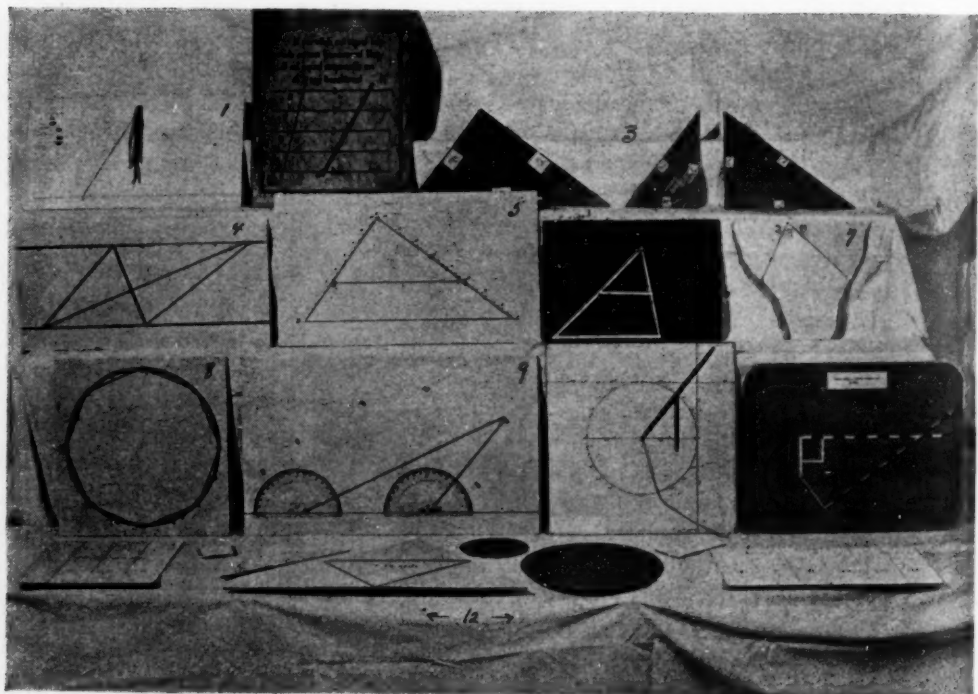


PLATE II

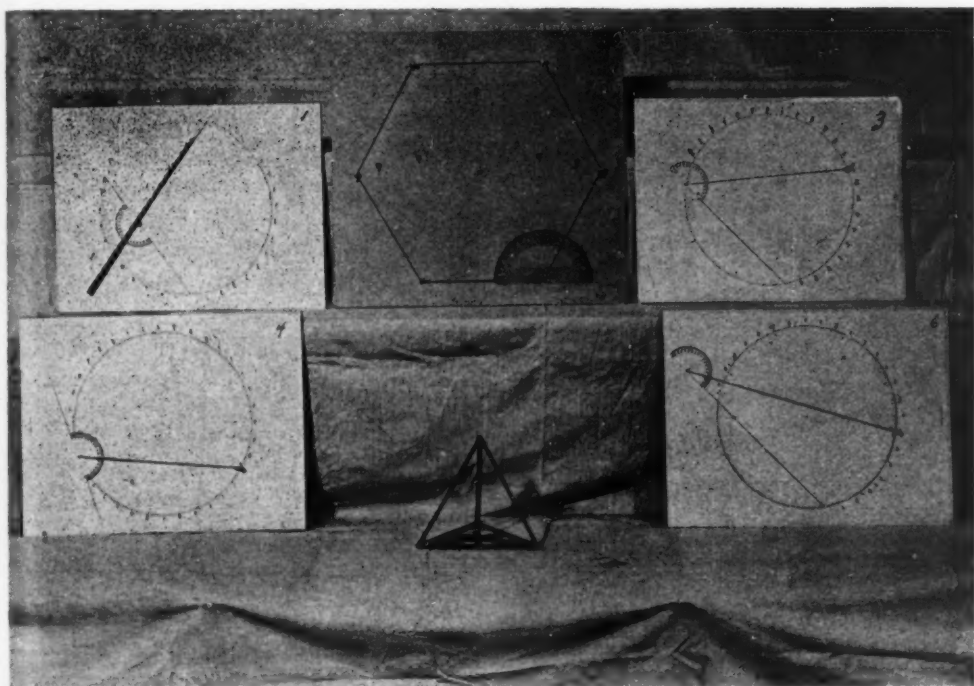


PLATE III

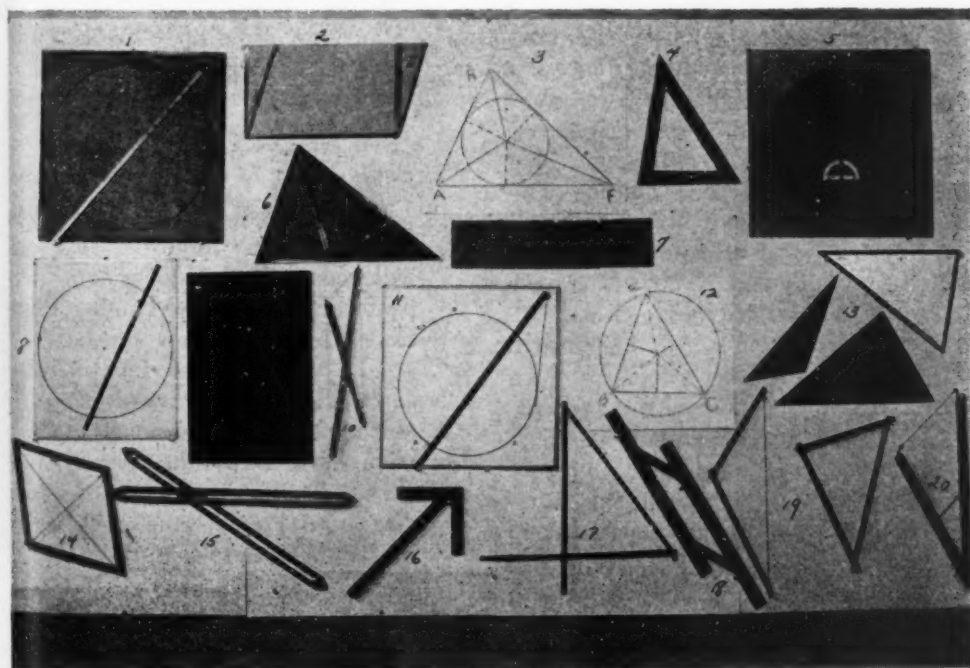


PLATE IV

Two models, Plate III-6, are given to the class with directions to find out what it thinks is the relationship between an angle formed by two secants and its intercepted arcs. One circle is marked at ten degree intervals.

The data from one board are

Arcs		Angle
50°	10°	20°
100°	20°	40°
140°	30°	55°
180°	50°	65°

From the second board

Arcs		Angle
25°	5°	10°
35°	7°	14°
70°	13°	29°
195°	55°	70°

The class will be about equally divided as to whether the conclusion is one-third the sum or one-half the difference of the arcs. Upon testing we find that each of these conclusions works in the first three cases of the first group. In the fourth case, however, one-half the difference does give 65°, but one-third the sum gives 76 $\frac{2}{3}$ °. We do allow for inaccuracies in the models but the pupils know that such a big difference is not due to inaccuracy. Likewise, both conclusions check in the first three cases read from the second board, while in the fourth case the one-third the sum is 13 $\frac{1}{3}$ ° off. Again this is too large a difference to be charged to inaccuracy.

Here is an opportunity to show the danger of drawing an inductive conclusion from too few cases. Of course the pupils realize that in this situation we are really not entitled to any conclusion and that the only way of testing their "one-half the sum" is to see if we can prove it deductively. This we do immediately. The writer feels that some pupils have never been really convinced of the advan-

tage and the inclusiveness of a deductive proof until this lesson is studied.

#### V. *Suggestions for the proof of a theorem.*

1. Plate II-7. When the angles B and C are removed from the base and taped to the vertex, the pupil not only sees that the sum of the angles is 180° but also has a hint as to how the construction line for the proof could be drawn.
2. Plate IV-2. In developing the area of a parallelogram theorem, if triangle 1 is kept bent back out of sight and then appears at the right moment in the discussion, a clue is given to the construction lines and to the method of proof.
3. Plate IV-6. By proper bending of the cardboard, the pupil can be led to use the method of analysis until he gets back to the auxiliary lines for the proof of this inequality theorem.
4. Most of the class can work out the steps for circumscribing a circle about, or inscribing a circle in a triangle by using Plate IV-3, 12 and two simple locus theorems. The use of various colors for the lines is important here.

#### VI. *Rapid review of certain relationships.*

If a pupil is stalled on a problem because he has forgotten that the diagonals of a rhombus are perpendicular he can be handed Plate IV-14. A rapid review can be built around the models so that the class again gets a visual impression of each fact reviewed. This is much quicker and more dramatic than merely stating the theorem or using blackboard diagrams. Pieces which especially lend themselves to this are

1. Position of an altitude in various kinds of triangles. Plate I-18.
2. Position of a median in various kinds of triangles. Plate I-20.
3. Position of the center of a circle

circumscribed around different kinds of triangles. Plate I-16.

4. Angles of regular polygons. Plate III-2.
5. Areas of similar figures are to each other as the squares of corresponding lines. Plate II-12.
6. Meaning of  $c^2 = a^2 + b^2$ . Plate I-9.
7. Median, altitude, and angle bisector. Plate I-17.
8. Diagonals of the parallelograms. Plate I-10 and Plate IV-14.
9. Diameter perpendicular to a chord and its corollaries. Plate IV-5.
10. Triangles equal in area. Plate II-4.
11. 3-4-5 right triangle. Plate I-8.
12. Angle inscribed in a semicircle. Plate I-19.

VII. *To convince the student of the truth of a theorem.*

Teachers know that in a deductive system the "truth" of a theorem does not enter the picture and that a conclusion reached by correct deduction must be accepted. This is a difficult concept for a fourteen or fifteen year old mind. After the pupils have developed a body of theorems, they like to have them work. Dwell upon the meaning of "The product of the

segments of a chord passing through a fixed point in a circle is constant." What class hasn't some members brave enough to say that they don't believe it? Then they test it by measurement and are likely to get lost in the difficult fractions which their measurements will almost certainly involve. The use of Plate IV-8 avoids this. Several cases using only integers can be set up so that the pupil discovers the relationship from the model and then proves it.

Plate IV-1. The product of the whole secant times its external segment.

Plate IV-11. The tangent-secant proportion.

None of the foregoing lists is complete and it is readily seen that many of the models are used in several ways. They are not a substitute for the work but rather aids for the understanding of principles.

It is generally agreed that one of the primary purposes of geometry is the teaching of deduction, but that it is neither necessary nor desirable to deduce every relationship which we study in the course. Teachers who have a large body of subject matter to cover in a limited time would find that the use of models allows for acceleration of certain parts of the work without a sacrifice of understanding.

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# EDITORIAL

## David Eugene Smith

DAVID EUGENE SMITH, professor emeritus of mathematics at Teachers College, Columbia University since 1926, died at his home in New York City on Saturday, July 29, 1944, at the age of eighty-four.

He was born in Cortland, N. Y., the son of Abram P. and May Elizabeth Bronson Smith. Dr. Horace Bronson, a learned country doctor, taught his daughter, May Elizabeth, nature study, science, Latin, and Greek while she was still very young, and she in turn instructed her son David Eugene along these lines. As a result, he could speak both Latin and Greek as a boy, and much preferred studying the Classics to weeding the garden or other chores assigned him by his father.

When the Cortland State Normal School opened, David Eugene was the first student to enroll, and at seventeen years of age he entered Syracuse University. While in Syracuse he studied art and languages, especially Hebrew. As a result, he could read the Bible in its original language, and he once expressed the opinion, "If you want to find the beauty of the Bible, read it in the Hebrew tongue, especially the Book of David." He held the degrees of Ph.M., Ph.D., and LL.D. from Syracuse University, Master of Pedagogy from Michigan State Normal College, D.Sc., from Columbia University, and L.H.D. from Yeshiva College.

After his graduation from Syracuse, in 1881, David Eugene Smith studied law in his father's office, and was admitted to the bar in 1884—the youngest lawyer in Cortland. In 1884 he began teaching mathematics at the State Normal School in Cortland. Seven years later he became Professor of Mathematics at the Michigan State Normal College at Ypsilanti. After three years as principal of the New York State Normal School at Brockport, Dr.

Smith became Professor of Mathematics at Teachers' College in 1901.

In 1887 Dr. Smith married Fannie Taylor, who died in 1928. He later married Eva May Luce, who survives him. He is also survived by his sister, Mrs. A. M. Jewett, and a niece, Mrs. Helen Jewett McAleer, both of Cortland.

Dr. Smith was the author of many mathematics textbooks in wide use in this country, books on the history and teaching of mathematics and books of more popular appeal, such as *Number Stories of Long Ago*, which was the outgrowth of stories he told nightly to children where he spent his summer vacation many years ago. In 1933 he was decorated by the late Reza Khan Pahleri, then Shah of Iran, for his translation of *The Rubaiyat of Omar Khayyam*. Dr. Smith was also the author of numerous articles published in mathematical and educational journals. Famous among these articles was "Religio Mathematici," which appeared in the November, 1921, issue of *Teachers College Record*.

Other works by Dr. Smith are "History of Modern Mathematics," "Rara Arithmetica," "Teaching of Arithmetic," "History of Japanese Mathematics," "Our Debt to Greece and Rome in Mathematics" and "Historical-Mathematical Paris."

An authority on the history of mathematics, Dr. Smith spent many years of his life collecting rare books and manuscripts abroad, both for himself and for the late George Plimpton of Ginn and Company, textbook publishers. All of Dr. Smith's valuable collection and that of Mr. Plimpton were recently given to Columbia University and are now housed in Low Memorial Library.

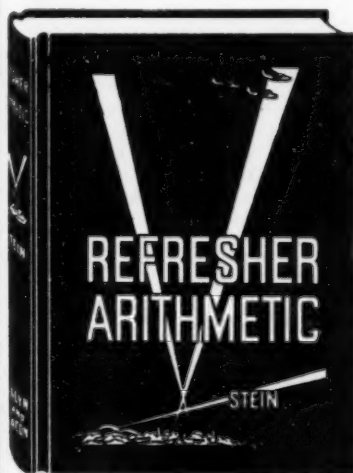
His love of travel (he made eighty Atlantic crossings), his interest in acquiring

rare books and beautiful art pieces, and his willingness to share all these with his friends made him an ideal host and his home a center of culture. Miss Bertha M. Frick, curator of the David Eugene Smith Library, quotes him as saying once, "I love to sit here and let my eyes wander. Wherever they fall I love again my experience in finding that particular treasure—on a trip down a river infested by crocodiles; sitting cross-legged in the mud with a native chief; wandering among the ruins of an ancient Buddhist monastery, ghostly in the moonlight."

David Eugene Smith was a very active member of most of the well-known mathe-

matical organizations. He was a former mathematics editor of the New International Encyclopedia, the Encyclopaedia Britannica, Monroe's Cyclopedia of Education and the New Practical Reference Library. He was an associate editor of "The American Mathematical Monthly," and honorary president of the International Commission on the Teaching of Mathematics. His wealth of knowledge was gleaned in many countries and in many languages. A scholar and a great teacher, he was loved and revered by all who had the good fortune to know him. We shall not see his like soon.

W. D. R.



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# ◆ IN OTHER PERIODICALS ◆

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Midwood High School, Brooklyn, New York

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1. Brink, R. W., "College Mathematics During Reconstruction," pp. 61-74.
2. McBrien, V. O., "Cardioids Associated with a Cyclic Quadrangle," pp. 74-78.
3. Derry, Douglas, "Affine Geometry of Convex Quartics," pp. 78-83.
4. Brand, Louis, "The Eight-Point Circle and the Nine-Point Circle," pp. 84-87.
5. "War Information: Notes on The Navy V-12 Program; The Training Program in Meteorology; The Alien Book Republication Program; Mathematics and the New Selective Service Regulations."

March 1944, Vol. 51, No. 3.

1. Wade, T. L., and Bruck, R. H., "Types of Symmetries," pp. 123-129.
2. Stabler, E. R., "Boolean Representation Theory," pp. 129-132.
3. Beiler, A. H., "An Electrical Chinese Ring Puzzle," pp. 133-137.
4. Schwerdtfeger, H., "Skew-Symmetric Matrices and Projective Geometry," pp. 137-148.
5. Dehn, Max, "Mathematics, 200 B.C.-600 A.D."
6. Dennis, F. L., "A Slide Rule Solution of Oblique Spherical Triangles," pp. 159-161.
7. Bellman, Richard, "A Note on The Product of Linear Forms," pp. 161-162.
8. "War Information: The Republication of Foreign Mathematical Tables; Quotas in Military Training Programs; Central Clearing Agency for Accreditation; Enlisted Men Entering the Navy V-12 Program; The Navy V-5 Program."

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1. Synge, J. L., "Focal Properties of Optical and Electromagnetic Systems," pp. 185-200.
2. Ayres, W. L., "Interesting The Engineering Student," pp. 200-205.
3. Parker, W. V., and Pryor, J. E., "Polygons of Greatest Area Inscribed in an Ellipse," pp. 205-209.
4. Moulton, E. J., (a) "A Speed Test Question, A Problem in Geography," p. 216; (b) "Comments on the Problem in Geography," p. 220.
5. Durham, R. L., "A Simple Construction for the Approximate Trisection of an Angle," pp. 217-218.
6. Wayne, Alan, "A Table for Computing Perimeters of Ellipses," pp. 219-220.
7. "War Information: Deferment of Instructors in the Navy College Program; Description of the Profession of Mathematics; From the National Roster of New Selective

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1. Smith, C. D., "Henry Lewis Rietz: 1875-1943," pp. 182-184.
2. Springer, C. E., "Evaluation of a Limit by a Sequence of Triangles," pp. 185-187.
3. Bell, E. T., "Gauss and The Early Development of Algebraic Numbers," pp. 188-204.
4. Read, Cecil B., "Random Jottings from an Instructor's Notebook," pp. 205-211.

March 1944, Vol. 18, No. 6.

1. Sanders, S. T., "The William Betz Analysis," p. 218.
2. Bell, E. T., "Gauss and The Early Development of Algebraic Numbers," (concluded) pp. 219-233.
3. Zant, James H., "Teaching Mathematics to the A.A.F. College Training Detachment (Air Crew)," pp. 234-242.

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1. Sanders, S. T., "To Our Subscribers and The General Mathematical Public," pp. 258-260.
2. Miller, G. A., "An Eighth Lesson in The History of Mathematics," pp. 261-270.
3. Niessen, A. M., "On the Summation of Certain Types of Finite Series," pp. 271-275.
4. Sullivan, Sister Helen, "Mathematics in The Open Forum," pp. 276-279.
5. Ingersol, Benham M. (Captain A.U.S.), "Geometric Derivation of the Formula for Integration by Parts," pp. 280-283.

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1. Massey, Emil L., "A Challenge," p. 295.
2. Read, Cecil B., and Hanna, J. Ray, "The Interpretation of Certain Notations in Decimals," pp. 307-308.
3. Buchman, Aaron, "Generating Some Higher Plane Curves," pp. 309-314.
4. Nyberg, Joseph A., "Notes from a Mathematics Class Room," (continued) pp. 319-322.
5. Potter, Mary A., "The Mathematics Laboratory," pp. 367-373.
6. Read, Cecil B., "What is a Meridian?" pp. 379-380.

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1. Tate, M. W., "Notes on Approximate Computation," pp. 425-431.
2. Burg, Walter V., "An Experimental Construction of the Sine Curve," pp. 467-468.

3. Nyberg, Joseph A., "Notes from a Mathematics Class Room," (continued) pp. 469-472.

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2. Bickel, M. K., "Few Suggestions for The Teaching of Arithmetic," *Ohio Schools*, 22: 115+, March 1944.
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24. Sullivan, H., "Is Mathematics a Liberal Art or a Lost Art?" *Catholic Educational Review* 42: 222-227, April 1944.
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## THE FROST IS ON THE PUMPKIN

The husky rusty rustle of the tassels of the corn,  
 And the raspin' of the tangled leaves, as golden as the morn;  
 The stubble in the furries—kind o' lonesome-like, but still  
 A-preachin' sermons to us of the barns they grewed to fill;  
 The straw-stack in the medder, and the reaper in the shed;  
 The hosses in their stalls below, the clover overhead,  
 Oh, it sets my heart a'clickin' like the tickin' of a clock,  
 When the frost is on the pumpkin and the fodder's in the shock!

—JAMES WHITCOMB RILEY

# NEWS NOTES

## AMERICAN EDUCATION WEEK 1944

"Education for New Tasks" is the theme for the twenty-fourth annual observance of American Education Week.

The United States is engaged in the greatest war in history. Before us loom the tasks of the postwar years which only an educated citizenry can hope to master. Such times require a great public school system, excelling by far anything that we have yet accomplished in the education of our children, youth, and adults.

Education has made and is making an indispensable contribution to the winning of the war. Its role in the peace will be equally significant if the American people fully understand the potential power of education.

How can we win the peace? How can we maintain full employment? How can we combat intolerance? How can we conserve and improve our human resources? There are many factors in the solution of these momentous issues that will face the nation in the postwar years, but universal and adequate education of all the people is the basic ingredient of every sensible prescription for these problems.

We spare no expense to get people ready to win a war. Why? Because we know that only a trained people can win. Public sentiment would not tolerate for a moment any proposal to send American boys into battle without the best of training under the best instructors and with the best equipment that money can buy. Shall we do less to help our young people win the battles of the peace to come?

American Education Week is an opportunity to interpret the role of education in the postwar years as well as the present contribution of the schools to the war effort.

The NEA has prepared materials to assist local schools in the observance of American Education Week such as a poster, leaflets, a sticker, a manual, plays, a movie trailer, radio scripts, newspaper advertising mats, and other materials. Address the National Education Association, 1201 Sixteenth Street, N.W., Washington 6, D. C. for an order form and further information.

The following letter sent out by one of our State Representatives represents the type of work these members are doing. More power to them.—Editor.

Middlebury, Vermont  
May 1, 1944

To Mathematics Teachers in the State of Vermont:

The purpose of this note is to remind you of the opportunity that is yours to join the crusade for better teaching of mathematics which is sponsored by The National Council of Teachers of Mathematics. This is the only organization in the United States devoted exclusively to the teaching of mathematics in the elementary and secondary schools. It is an organization of and for mathematics teachers, controlled by them, and I am urging you all to support it and to

profit by the inspiring articles appearing in *THE MATHEMATICS TEACHER*.

No teacher of mathematics can invest two dollars that will bring greater returns than will his membership fee in The National Council. The following list of titles of articles recently appearing in *THE TEACHER*, gives some indication of the helpful suggestions available to its readers, timely suggestions which not only enhance one's appreciation and fascination for mathematics itself but also afford us countless devices and techniques whereby we can make our subject more endurable in the classroom:

Mathematics That Functions in War and Peace  
How Shall Geometry Be Taught  
Mathematics and the Armed Forces Educational Program  
A Grooved Mechanism  
Arithmetic Attitudes  
Snow White and the Seven Dwarves  
Early American Geometry  
Horner's Method and the Algorithm for Extraction of Square and Cube Roots  
Remedial Arithmetic in the Senior High School  
Developing of Interest and Skill in Handling Trinomial Squares  
Mathematical Theory of the Mercator Map  
It All Began Last Winter

—and many others.

I am enclosing a membership blank which I hope you will fill out and return either to me or to the office of the National Council in New York. I shall be happy to send you a sample copy of *THE MATHEMATICS TEACHER* upon request. The number of members in Vermont is disappointingly small. Let's double it this spring.

Faithfully yours,  
John G. Bowker  
Vermont State Representative  
The National Council of  
Teachers of Mathematics

The eighth meeting of the Men's Mathematics Club of Chicago was held on May 19, 1944. Professor L. R. Ford of the Illinois Institute of Technology spoke on "General Slide Rules" and Professor M. A. Sadowsky of the same Institute spoke on "Models in Mechanics."

The following officers were elected to serve for 1944-1945: Glenn F. Hewitt, *President*, Von Steuben High School, Chicago, Ill.; W. W. Barczewski, *Secretary-Treasurer*, Waukegan Township High School, Waukegan, Ill.; Glenn Anderberg, *Recording-Secretary*, Waukegan Township High School, Waukegan, Ill.

The Annual Spring Meeting of the Rhode Island Mathematics Teachers Association was held at Brown University in Providence on March 18, 1944. Dr. Chas. H. Smiley, Director of Ladd Observatory at Brown spoke on "Emergency Navigation with Minimum Equip-

ment." M. L. Herman of the Moses Brown School is President, and Harriet C. Whitaker of the Lincoln School in Providence is Secretary-Treasurer.

Dr. John A. Swenson, who recently retired as head of the mathematics department of the Andrew Jackson High School in St. Albans, Queens, N. Y. died at his home, 16 Lancaster Avenue, Baldwin, L. I. on May 2, 1944. Dr. Swenson was 63 years old having been born in Sweden 63 years ago. He held the M.A. and Ph.D. degrees from Columbia University, was the author of a series of high school mathematics texts on "Integrated Mathematics," and was a well-known figure and leader among the mathematics teachers of New York City and vicinity. He was an indefatigable worker and was unusually enthusiastic about the subject which he taught and loved.

Dr. Swenson leaves a widow, the former Mary Kidd; two daughters, Mrs. Margaret S. Reynolds and Mrs. Esther S. Wallace; a son, John D. Swenson; four brothers, and four grandchildren.

A more complete story of Dr. Swenson's life and work appeared in the March 1944 issue of THE MATHEMATICS TEACHER.

A regional meeting of the National Council of Teachers of Mathematics was held at Wilson Teachers College, Washington, D. C., on Saturday, April 1, 1944.

## PROGRAM

### THEME:

LESSONS IN THE TEACHING OF MATHEMATICS WHICH ARE BEING LEARNED FROM THE WAR

### Mathematics Exhibit

There was an exhibit in Room 215 of mathematics teaching materials used by Washington teachers. Dr. Daniel B. Lloyd of Roosevelt High School was in charge.

### MORNING PROGRAM

10:00 A.M. General Meeting in Auditorium. Presiding: Veryl Schult, Head of Mathematics Dept. Div. 1-9, Washington, D. C.

The Work of the National Council  
Mrs. Ethel H. Grubbs, Head of Math.  
Dept. Div. 10-13, Washington, D. C.

The following speakers presented certain facts and problems and raised vital questions. Then in the afternoon session, these problems and questions were considered thoroughly in the discussion groups, after which the Recorder from each group reported back the important conclusions and recommendations to the general assembly.

1. THE IMPORTANCE OF MATHEMATICS TO YOUR U. S. NAVY

Speaker: Lieutenant Guy L. Bond, U. S. Navy

2. WILL THE EMPHASIS BE ON TECHNICAL OR ON SOCIAL IMPLICATIONS OF MATHEMATICS AFTER THE WAR?

Speaker: Francis R. Lankford, Director of Research, Richmond, Va. Public Schools

3. THE INTEREST IN MATHEMATICS AS SHOWN IN THE ARMED FORCES EDUCATIONAL SERVICES

Speaker: Capt. S. E. Burr, A.G.D. Army Education Branch, Morale Services Division, A.S.F.

4. HAVE WE LEARNED ANYTHING FROM THE ARMED FORCES TO GUIDE TEACHERS OF ARITHMETIC IN GRADES 4, 5, AND 6?

Speaker: Dr. John R. Clark, Teachers College, Columbia University

5. WHAT GUIDANCE FOR MATHEMATICS DOES THE ARMY PREINDUCTION UNIT PROVIDE?

Speaker: Major Ralph C. Winrich, U. S. Army

6. SHOULD THE EMPHASIS ON AVIATION IN MATHEMATICS CONTINUE?

Speaker: Bruce Uthus, Director of Aviation Education Service, C. A. A.

7. HOW CAN WE PROVIDE BETTER VISUAL AIDS?

Speaker: Lieutenant Francis Noel, U. S. Navy

8. THE COUNCIL'S COMMITTEE ON POST WAR PLANNING OF MATHEMATICS PROGRAMS

Speaker: Dr. Raleigh Schorling, Consultant, U. S. Navy

### AFTERNOON PROGRAM

2:30-3:30 Discussion groups with the speakers of the morning session.

1. THE IMPORTANCE OF MATHEMATICS TO YOUR NAVY

Room 205

Chairman: Mrs. Evelyn Lego, Langley Junior High School.

Recorder: Mrs. Nanette R. Blackiston, Supervisor of Mathematics, Baltimore, Md.

Consultant: Lieutenant Guy L. Bond.

2. WILL THE EMPHASIS BE ON TECHNICAL OR ON SOCIAL IMPLICATIONS OF MATHEMATICS AFTER THE WAR?

Room 201

Chairman: Mary Maciulla, Gordon Junior High School.

Recorder: Lela Lynam, Supervisor of Mathematics, Wilmington, Delaware.

Consultant: Dr. Francis R. Lankford.

3. ARMED FORCES EDUCATIONAL SERVICES. No discussion group.

4. HAVE WE LEARNED ANYTHING FROM THE ARMED FORCES TO GUIDE TEACHERS OF GRADES 4, 5, AND 6?

Room 203

Chairman: Agnes Motyka, Ludlow Elementary School.

Recorder: Mrs. Evelyn S. Boyer, Brightwood Elementary School.

Consultant: Dr. John R. Clark, Teachers College, Columbia University.

5. WHAT GUIDANCE FOR MATHEMATICS DOES THE ARMY PREINDUCTION UNIT PROVIDE?

Room 217

Chairman: Ethel Smith, Anacostia High School.

Recorder: Mrs. Gwendolyn D. Holland, Randall Junior High School.

Consultant: Major Ralph C. Winrich.

6. SHOULD THE EMPHASIS ON AVIATION IN MATHEMATICS CONTINUE?

Room 207

Chairman: Lee S. Gilbert, Coolidge High School.

Recorder: Carol V. McCamman, McKinley High School.

Consultant: Bruce Uthus.

### 7. HOW CAN WE PROVIDE BETTER VISUAL AIDS?

Room 215

Chairman: Dr. Daniel B. Lloyd, Roosevelt High School.

Recorder: Arthur D. Jewell, Armstrong High School.

Consultant: Lieutenant Francis Noel.

### 8. THE COUNCIL'S COMMITTEE ON POST WAR PLANNING OF MATHEMATICS PROGRAMS

Room 211

Chairman: Mrs. Gladys P. Payne, Garnet-Patterson Jr. H. S.

Recorder: Dr. Clyde M. Huber, Wilson Teachers College.

Consultant: Dr. Raleigh Schorling.

### 3:30-4:00—REPORTS OF DISCUSSION GROUPS BY RECORDERS

### 4:00-4:15—SUMMARY OF DAY'S ACCOMPLISHMENTS by Dr. Raleigh Schorling.

There were two mathematics movies: "Mysteries of Snow" (Silent, 6 minutes); "Origins of Mathematics" (Sound, 10 minutes).

The 30th annual meeting of the Kansas Section of the Mathematical Association of America and the 40th annual meeting of the Kansas Association of Teachers of Mathematics was held at Washburn University in Topeka on April 15, 1944.

#### Morning Session

Paul Eberhart, Washburn, Presiding.

1. Algebraic Functions—D. H. Richert, Bethel College.
2. Mathematics in the Navy Training Programs—G. W. Smith, University of Kansas.
3. Mathematics in Pre-Radar Training—A. E. White, Kansas State College, Manhattan.
4. Mathematics in the Army Training Program—E. B. Stouffer, University of Kansas.
5. Mathematics in Manufacture of Aircraft—Edison Greer, Beech Aircraft Corporation.
6. Mathematics in the Army Air Force Program—O. J. Peterson, K. S. T. C., Emporia.
7. Placement Tests for Air Corps Students—C. B. Read, University of Wichita.
8. The Armed Forces Institute Tests—Laura Greene, Washburn University.
9. Correlation of Entrance Test Scores and Term Grades—W. T. Stratton in cooperation with J. C. Peterson, Kansas State College, Manhattan.

#### 12:30—Noon Luncheon

#### Afternoon Session

Herbert Bishop, Manhattan High School, Presiding.

1. Random Jottings from an Instructor's Notebook—C. B. Read, University of Wichita.
2. Pre-Induction Courses in Mathematics—Edna Austin, Topeka High School.
3. The Mathematics Program in the High School at Present and in the Post-War Period—T. J. LaRue, Junction City High School.
4. Report of the Committee on the Improvement of Instruction—Gilbert Ulmer, University of Kansas.

#### Business Meetings

##### OFFICERS KANSAS SECTION M. A. A.

##### Chairman

Paul Eberhart, Washburn University, Topeka

##### Vice Chairman

Edison Greer, Beech Aircraft Corporation, Wichita

##### Secretary

Anna Marm, University of Kansas, Lawrence

##### Nominating Committee

W. T. Stratton, Kansas State College, Manhattan

O. J. Peterson, J. S. T. C., Emporia

Wealthy Babcock, University of Kansas, Chairman

##### OFFICERS K. A. T. M.

##### President

Herbert Bishop, Manhattan High School

##### Vice President

Sara Belle Wasser, Pratt High School

##### Secretary-Treasurer

Martha Rayhill, Lawrence High School

##### Editor Bulletin

Ina E. Holroyd, K. S. C., Manhattan

##### Nominating Committee

Lowell Bailey, Lawrence High School

Bernice Boyle, Topeka High School

Chairman to be announced

Association of Mathematics Teachers of New Jersey's seventy-ninth regular meeting was held at Essex House, Newark, New Jersey on Saturday April 29, 1944.

#### GENERAL TOPIC

##### Mathematics for Wartime and for Peacetime

#### MORNING SESSION

*President's Message*—Present Trends in Mathematics. *Dr. David R. Davis*, President of the Association of Mathematics Teachers of New Jersey

##### Panel Discussion—

*Theme*—The Mathematics of our Secondary Schools

*Presiding*—*Dr. David R. Davis*, President of the Association of Mathematics Teachers of New Jersey

*Speakers*—*Dr. Amanda Loughren*, Supervisor of Mathematics, Public Schools, Elizabeth

*Mr. J. Dwight Daugherty*, Head of Mathematics Department, Eastside High School, Paterson

*Dr. Carl N. Shuster*, Professor of Mathematics, State Teachers College, Trenton

*Mr. John W. Colliton*, Head of Mathematics Department (Retired), Central High School, Trenton

##### General Discussion—

##### Annual Business Meeting—

#### LUNCHEON

*Joint Session*—Association of Mathematics Teachers of New Jersey and the New Jersey Elementary Mathematics Association

*Presiding*—*Dr. David R. Davis*, President of the Association of Mathematics Teachers of New Jersey

*Guest Speaker—Dr. William G. Betz, Specialist in Mathematics for the Public Schools of Rochester, New York*  
**Topic—Whither Mathematics?**

#### ASSOCIATION OFFICERS

**President**—Dr. David R. Davis, State Teachers College, Montclair  
**Vice Presidents**—J. Dwight Daugherty, Eastside High School, Paterson; Madeline Messner, Abraham Clark High School, Roselle; Dr. Fred L. Bedford, State Teachers College, Jersey City  
**Secretary-Treasurer**—Mary C. Rogers, Roosevelt Junior High School, Westfield  
**Corresponding Secretary**—Dorothy DeWitt, Regional High School, Springfield  
**Recording Secretary**—Mollie R. Bruce, Princeton High School, Princeton

#### COMMITTEES

**Committee on Arrangements**—Carl N. Shuster, Madeline Messner, Mollie R. Bruce  
**Program Committee**—Mary C. Rogers, Mollie R. Bruce, J. Dwight Daugherty, Fred L. Bedford, David R. Davis  
**Committee on Membership**—Mary C. Rogers, Dorothy De Witt, Madeline Messner, Dorothy Frapewell, Florence Gorgens  
**Committee on Bulletin**—Dr. Howard F. Fehr, J. Dwight Daugherty, Dorothy DeWitt, Madeline Messner, Mary C. Rogers, Mollie R. Bruce

The Third Annual Meeting of the Mathematical Association of America, Metropolitan New York Section was held at New York University, Saturday, April 22, 1944.

#### PROGRAM

Morning Session, 9:30 A.M.,  
 Room 703, Main Building

**Chairman:** Professor R. M. Foster, Polytechnic Institute of Brooklyn

#### ADDRESS OF WELCOME

**Dean Charles M. McConn,** Washington Square College, New York University

#### ELEMENTARY MATHEMATICAL THEORY OF EXTERNAL BALLISTICS

**Professor Harris F. MacNeish,** Brooklyn College

#### APPLICATIONS OF MATHEMATICS IN AERONAUTICAL ENGINEERING

**Professor R. Paul Harrington,** Polytechnic Institute of Brooklyn

#### COMBINATORIAL STATISTICS

**Dr. Jacob Wolfowitz,** Columbia University

Afternoon Session, 2:00 P.M.,

Room 703, Main Building

**Chairman:** Mr. Max Peters, Long Island City High School

#### BUSINESS MEETING AND ELECTION OF OFFICERS FOR 1944-1945

#### A GUIDING PHILOSOPHY FOR TEACHING DEMONSTRATIVE GEOMETRY

**Mr. Morris Hertz,** Forest Hills High School

#### MATHEMATICS AND EMPIRICAL SCIENCE

**Professor Carl G. Hempel,** Queens College

*Officers of the Metropolitan New York Section 1943-1944*

**Ronald M. Foster,** Polytechnic Institute of Brooklyn, *Chairman*; **Max Peters,** Long Island City High School, *Vice-Chairman*; **Howard E. Wahlert,** New York University, *Secretary*, and **Frederic H. Miller,** Cooper Union, *Treasurer*.

The Alabama Branch of the National Council of Teachers of Mathematics met with the Alabama Educational Association in Birmingham on March 30, 1944.

#### PROGRAM

**Mr. J. Eli Allen** spoke on "The Work of the National Council of Teachers of Mathematics" and **Dr. J. M. Malone** spoke on "Modern War Weighs the Mathematics Teacher."

New officers elected were: *President*—**M. H. Pearson,** Montgomery; *Vice President*—**Miss Annie C. Marts,** Huntsville; *Secretary-Treasurer*—**Miss Laura K. Miller,** Birmingham.

The following program has been planned for the Mathematics Section of the Indiana State Teachers Association in the War Memorial Auditorium in Indianapolis on October 26, 1944.

#### 9:30 A.M.

1. Music—Girl's Glee Club, Manual Training High School, Indianapolis, directed by Frieda M. Hart.
2. Address—The Present Situation in Junior High School Mathematics—**Professor W. D. Reeve,** Teachers College, Columbia University.
3. Discussion.

#### 2:30 P.M.

1. Business.
2. Visual Education—Films, "Light on Mathematics," presented by the Jam Handy Corporation.

#### Officers

**President:** Ada M. Coleman, Indianapolis.

**Vice President:** Anton Wegoner, Aurora.

**Secretary:** Della M. Sanders, Frankfort.

The Mathematics Section of the Southeastern Ohio Education Association will meet on October 27 in Cincinnati. **Professor W. D. Reeve** of Teachers College will speak on "The Mathematics of the Junior High School."

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